INEQUALITIES 3

1. Solve the following inequalities

a) $x^2 + x - 6 \leq 0$

b) $x^2 + 9 > 0$

c) $x^2 - 5x < 0$

d) $-x^2 + 4x - 7 > 0$

e) $4x^2 - 4x + 1 > 0$

f) $-x^2 + 5x - 6 \geq 0$

g) $2x(x-1) - x + 1 > 0$

2. Solve the following systems of inequalities:

a) $\left.\begin{array}{l} 4x^2 - 16 < 0 \\ 2x - 3 \leq 5 \end{array}\right\}$

b) $\left.\begin{array}{l} x^2 + 3x - 4 \leq 0 \\ x - 2 \leq 2x + 1 \end{array}\right\}$

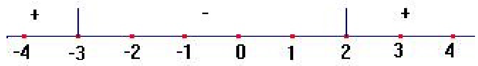
c) $\left.\begin{array}{l} 5x > x^2 + 4 \\ x + 4 \leq 2x - 1 \end{array}\right\}$

SOLUTION

a) $x^2 + x - 6 \le 0$ we start solving the equation:

$$x^2 + x - 6 = 0 \Rightarrow x = \frac{-1 \pm \sqrt{1+24}}{2} = \begin{cases} 2 \\ -3 \end{cases}$$

Studying the sign:



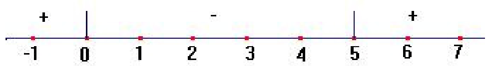
Solution:

$[-3,2]$

b) $x^2 + 9 > 0$ we cannot factorize this polynomial: $x^2 + 9 = 0 \Rightarrow x^2 = -9$
we see that $x^2 + 9$ is **always positive** , solution **R**.

c) $x^2 - 5x < 0$ solving: $x^2 - 5x = 0 \Rightarrow x(x-5) = 0 \Rightarrow \begin{cases} x = 0 \\ x = 5 \end{cases}$

studying the sign:



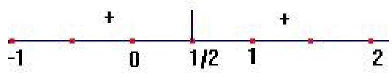
Solution :

$(0,5)$

d) $-x^2 + 4x - 7 > 0 \rightarrow x = \frac{-4 \pm \sqrt{16-28}}{-2}$ so, this polynomial don´t have any real root and it is always negative, so this inequality do not have any **solution**.

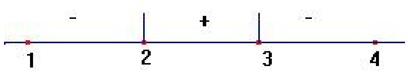
e) $4x^2 - 4x + 1 > 0$ $\quad 4x^2 - 4x + 1 = 0 \Rightarrow x = \frac{4 \pm \sqrt{16-16}}{8} = \frac{1}{2}$ studying the sign:

Solution : R



f) $-x^2 + 5x - 6 \ge 0$, solving the equation: $-x^2 + 5x - 6 = 0 \Rightarrow x = \frac{-5 \pm \sqrt{1}}{-2} = \begin{cases} 2 \\ 3 \end{cases}$

studying the sign:



Solution:

$[2,3]$

g) $2x(x-1) - x + 1 > 0 \Rightarrow 2x^2 - 2x - x + 1 > 0 \Rightarrow 2x^2 - 3x + 1 > 0$

solving, $2x^2 - 3x + 1 = 0 \Rightarrow x = \dfrac{3 \pm \sqrt{9-8}}{4} = \left\langle \begin{matrix} 1 \\ \dfrac{1}{2} \end{matrix} \right.$, sign:

Solution:

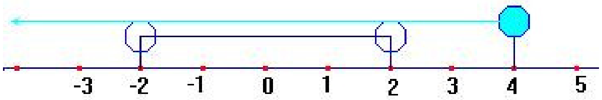
$\left(-\infty, \dfrac{1}{2}\right) \cup (1, +\infty)$

2. Solve the following systems of inequalities:

a) $\left.\begin{matrix} 4x^2 - 16 < 0 \\ 2x - 3 \le 5 \end{matrix}\right\} \rightarrow 2x \le 8 \rightarrow x \le 4 \Rightarrow Sol: [4, +\infty)$

$4x^2 - 16 = 0 \rightarrow 4x^2 = 16 \rightarrow x^2 = 4 \rightarrow x = \pm 2$

Studying the sign: Sol: $(-2, 2)$



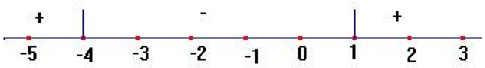
Solution of the system: $(-2, 2)$

b) $\left.\begin{matrix} x^2 + 3x - 4 \le 0 \\ x - 2 \le 2x + 1 \end{matrix}\right\} \rightarrow x - 2x \le 1 + 2 \rightarrow -x \le 3 \rightarrow x \ge -3 \Rightarrow Sol: [-3, +\infty)$

$x^2 + 3x - 4 = 0 \rightarrow x = \dfrac{-3 \pm \sqrt{9+16}}{2} = \left\langle \begin{matrix} 1 \\ -4 \end{matrix} \right.$

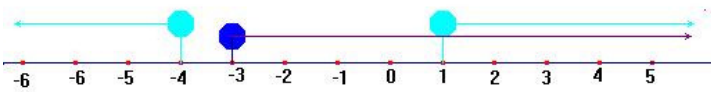
Studying the sign:

Sol: $(-\infty, -4] \cup [1, +\infty)$



Solution of the system:

$[1, +\infty)$

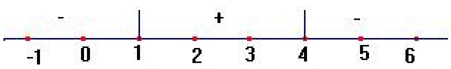


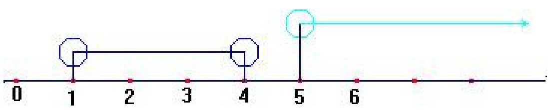
c) $\left.\begin{matrix} 5x > x^2 + 4 \\ x + 4 \le 2x - 1 \end{matrix}\right\} \rightarrow x - 2x \le -1 - 4 \rightarrow -x \le -5 \rightarrow x \ge 5$

$5x - x^2 - 4 > 0 \Rightarrow -x^2 + 5x - 4 > 0 \Rightarrow x = \dfrac{-5 \pm \sqrt{25-16}}{-2} = \left\langle \begin{matrix} 1 \\ 4 \end{matrix} \right.$

Studying the sign:

Sol: $(1, 4)$





Solution of the system: ϕ