

CONTROL INTEGRALES INDEFINIDAS

1) $\int (x^2 + 7x - 5) \cdot \cos x \, dx$

2) $\int x \operatorname{tg}^2 x \, dx$

3) $\int \frac{\ln^2 x}{x^2} \, dx$

4) $\int \frac{dx}{2\operatorname{sen}^2 x + 3\operatorname{cos}^2 x}$

5) $\int \frac{e^x + e^{2x} + e^{3x}}{e^{4x}} \, dx$

6) $\int \frac{2x^2 + x - 1}{3x(x+2)(x-2)} \, dx$

7) Sea $f : (-1,1) \rightarrow \mathbb{R}$ la función definida por $f(x) = \operatorname{Ln}(1-x^2)$. Calcula la primitiva de f cuya gráfica pasa por el punto $(0,1)$

SOLUCIONES

$$\begin{aligned}
 1) \int (x^2 + 7x - 5) \cdot \cos x \, dx &= \left(\text{llamamos } \begin{cases} u = x^2 + 7x - 5 \\ dv = \cos x \, dx \end{cases} \left\{ \begin{array}{l} du = (2x + 7) dx \\ v = \text{sen } x \end{array} \right\} \right) = \\
 &= (x^2 + 7x - 5) \text{sen } x - \int (2x + 7) \text{sen } x \, dx = \left(\text{haciendo } \begin{cases} u = 2x + 7 \\ dv = \text{sen } x \, dx \end{cases} \left\{ \begin{array}{l} du = 2 dx \\ v = -\cos x \end{array} \right\} \right) \\
 &= (x^2 + 7x - 5) \text{sen } x + (2x + 7) \cos x - 2 \int \cos x \, dx = \\
 &= (x^2 + 7x - 7) \text{sen } x + (2x + 7) \cos x + C
 \end{aligned}$$

$$\begin{aligned}
 2) \int x \, \text{tg}^2 x \, dx &= \left(\begin{cases} u = x \\ dv = \text{tg}^2 x \, dx \end{cases} \left\{ \begin{array}{l} du = dx \\ v = \text{tg } x - x^{(*)} \end{array} \right\} \right) = \\
 \text{(*)} v &= \int \frac{\text{sen}^2 x}{\cos^2 x} \, dx = \int \frac{1 - \cos^2 x}{\cos^2 x} \, dx = \int \frac{1}{\cos^2 x} \, dx - \int 1 \, dx = \text{tg } x - x \\
 &= x \text{tg } x - x^2 - \int (\text{tg } x - x) \, dx = x \text{tg } x - \frac{x^2}{2} - \ln(\cos x) + C
 \end{aligned}$$

$$3) \int \frac{\ln^2 x}{x^2} \, dx = \left(\begin{cases} u = \ln^2 x \\ dv = \frac{1}{x^2} \, dx \end{cases} \left\{ \begin{array}{l} du = \frac{2 \ln x}{x} \, dx \\ v = -\frac{1}{x} \end{array} \right\} \right) = -\frac{1}{x} \ln^2 x + 2 \int \frac{\ln x}{x^2} \, dx = \text{(y ahora,}$$

$$\left. \begin{array}{l} u = \ln x \\ dv = \frac{1}{x^2} \, dx \end{array} \right\} \left\{ \begin{array}{l} du = \frac{dx}{x} \\ v = -\frac{1}{x} \end{array} \right\} = -\frac{1}{x} \ln^2 x + 2 \left[-\frac{\ln x}{x} + \int \frac{dx}{x^2} \right] = -\frac{1}{x} (\ln^2 x + 2 \ln x + 2) + C$$

$$\begin{aligned}
 4) \int \frac{dx}{2 \text{sen}^2 x + 3 \cos^2 x} &= \text{(Dividimos numerador y denominador entre } 3 \cos^2 x) = \\
 &= \int \frac{\frac{1}{3 \cos^2 x} dx}{\frac{2 \text{sen}^2 x}{3 \cos^2 x} + 1} = \frac{1}{3} \int \frac{\frac{1}{\cos^2 x} dx}{\left(\frac{\sqrt{2}}{\sqrt{3}} \text{tg } x \right)^2 + 1} = \left(\begin{cases} t = \frac{\sqrt{2}}{\sqrt{3}} \text{tg } x \\ dt = \frac{\sqrt{2}}{\sqrt{3}} \frac{1}{\cos^2 x} \end{cases} \right) = \frac{1}{\sqrt{6}} \text{arctg} \left(\frac{\sqrt{2}}{\sqrt{3}} \text{tg } x \right) + C
 \end{aligned}$$

$$\begin{aligned}
 5) \int \frac{e^x + e^{2x} + e^{3x}}{e^{4x}} \, dx &= \int e^{-3x} \, dx + \int e^{-2x} \, dx + \int e^{-x} \, dx = \\
 &= -\frac{1}{3} e^{-3x} - \frac{1}{2} e^{-2x} - e^{-x} + C
 \end{aligned}$$

$$6) \int \frac{2x^2 + x - 1}{3x(x+2)(x-2)} dx = \int \frac{1/4}{3x} dx + \int \frac{5/24}{x+2} dx + \int \frac{9/24}{x-2} dx =$$

$$\frac{1}{12} \ln|x| + \frac{5}{24} \ln|x+2| + \frac{9}{24} \ln|x-2| + C$$

7) Empezamos calculando $\int \ln(1-x^2) dx$ que se hace por partes:

$$\left. \begin{array}{l} u = \ln(1-x^2) \\ dv = dx \end{array} \right\} \begin{array}{l} du = \frac{-2x}{1-x^2} dx \\ v = x \end{array} \Rightarrow \int \ln(1-x^2) dx = x \ln(1-x^2) - \int \frac{-2x^2}{1-x^2} dx =$$

$= x \ln(1-x^2) + 2 \int \frac{x^2}{1-x^2} dx$ para hacer esta integral, dividimos:

$$\frac{x^2}{-x^2+1} \frac{-x^2+1}{-1} \Rightarrow \frac{x^2}{1-x^2} = \frac{-1(1-x^2)+1}{1-x^2} = -1 + \frac{1}{1-x^2}$$

Con lo que tenemos $\int \frac{x^2}{1-x^2} dx = \int -1 dx + \int \frac{1}{1-x^2} dx = -x + \int \frac{A}{1-x} dx + \int \frac{B}{1+x} dx$

$$\frac{1}{1-x^2} = \frac{A}{1-x} + \frac{B}{1+x} \Rightarrow 1 = A(1+x) + B(1-x), \text{ sustituimos para } x=1 \text{ y } x=-1 \text{ y}$$

tenemos que $A = \frac{1}{2}$ y $B = \frac{1}{2}$, de donde, nuestra integral, nos quedará:

$$\int \ln(1-x^2) dx = x \ln(1-x^2) + 2 \left[-x + \frac{1}{2} \ln|1-x| + \frac{1}{2} \ln|1+x| \right] + C, \text{ es decir:}$$

$$\int \ln(1-x^2) dx = x \ln(1-x^2) - 2x + \ln|1-x^2| + C \text{ y, como la gráfica de la primitiva}$$

pedida pasa por (0,1), tendremos que: $0 - 0 + \ln 1 + C = 1 \Rightarrow C = 1$ por lo que la

$$\text{primitiva pedida es } x \ln(1-x^2) - 2x + \ln|1-x^2| + 1$$