EXAM 1_1 (Numbers/Percentages)

1) a) Which of the following numbers are rational numbers? (2 points)
   0.8383383383333333; 0.83; 0.8383; 0.83

   b) Arrange them in ascending order.
   c) Express each rational number as a fraction.
   d) Write three numbers between −0.83 and −0.83

2) Work out and simplify, writing the steps you have taken to reach the solution:
   (1.5 points)
   a) \(\frac{4}{3} \div \frac{2}{9} + 3^{-2} - \frac{9}{2} \div \frac{3}{7} = \)
   b) \(\left(\frac{1}{2} - 1\right)^2 \cdot \frac{6}{5} \div \frac{3}{2} \cdot \left(\frac{2}{5} - \frac{1}{2}\right) = \)

3) Write the following expressions as a single positive power, writing the steps you have taken to reach the solution:
   (1.5 points)
   a) \(\left(\frac{1}{2}\right)^{-3} \cdot \frac{4^{-2} \cdot 12^3 \cdot 8^2}{6^3 \cdot 16^3} = \)
   b) \(\left(\frac{3}{2}\right)^{-2} \cdot \left(\frac{9}{8}\right)^2 \cdot 12 = \)

4) There were twelve bananas on the counter. One-sixth of the bananas were eaten yesterday. Three-fourths of the bananas were eaten today. What fraction of the bananas have been eaten in all? (1 point)

5) 3/5 of the rabbits on the farm are grey, 2/3 of the rest are white, and the remaining rabbits, 20, are black. How many rabbits are on the farm? (1.25 points)

6) Martha bought a watch at 80% of the regular price. She paid €33.6 for the watch. What was the regular price? (1 point)

7) Tommy paid €21 for a book he purchased from a book club. This was a saving of €7 off the regular price. What percent discount did the book club give Tommy? (1.25 points)
1) a) Which of the following numbers are rational numbers?
   \[0.838383833\ldots; 0.83; 0.8383; 0.83\]
   Rational numbers: \(0.83; 0.8383; 0.83\)

   b) Arrange them in ascending order.
   \(0.83 < 0.8383 < 0.8383833\ldots < 0.83\)

   c) Express each rational number as a fraction.
   \[
   \begin{align*}
   0.83 & = \frac{83}{100} \\
   0.8383 & = \frac{8383}{10000}
   \end{align*}
   \]

   d) Write three numbers between \(-0.83\) and \(-0.83\)
   \(-0.83 < -0.833 < -0.832 < -0.831 < 0.83\)

2) Work out and simplify, writing the steps you have taken to reach the solution:

   a) \[
   \frac{4}{3} \div \frac{2}{9} + 3 \cdot \frac{9}{2} - \frac{3}{7} = \frac{4 \cdot 9}{3 \cdot 2} + \frac{1}{3} \cdot \frac{9}{2} - \frac{3}{7} = \frac{2^2 \cdot 3^2}{2 \cdot 3} + \frac{1}{9} - \frac{3^2 \cdot 7}{2 \cdot 3} = 6 + \frac{1}{9} - \frac{21}{2} = \frac{108}{18} + \frac{2}{18} - \frac{189}{18} = \frac{79}{18}
   \]

   b) \[
   \left( \frac{1}{2} - 1 \right) \cdot \frac{6}{5} - \frac{3}{2} \left( \frac{2}{5} - \frac{1}{2} \right) = \left( -\frac{1}{2} \right) \cdot \frac{6}{5} - \frac{3}{2} \cdot \frac{4 - 5}{10} = \frac{1}{4} \cdot \frac{6}{5} - \frac{3}{2} \cdot \left( -\frac{1}{10} \right) = \frac{6}{20} + \frac{3}{20} = \frac{9}{20}
   \]

3) Write the following expressions as a single positive power, writing the steps you have taken to reach the solution:

   a) \[
   \left( \frac{1}{2} \right)^3 \cdot \frac{4^2 \cdot 12^3 \cdot 8^2}{6^3 \cdot 16^3} = 2^3 \cdot \left( \frac{2^2}{2 \cdot 3} \right)^3 \cdot \left( \frac{3}{2^3} \right)^2 \cdot \left( \frac{2^5}{2^4} \right)^2 = 2^3 \cdot 2^{-4} \cdot 2^6 \cdot 3^3 \cdot 2^6 = \frac{2^{11}}{2^{15}} = \left( \frac{1}{2} \right)^4
   \]

   b) \[
   \left( \frac{3}{2} \right)^{-2} \cdot \left( \frac{9}{8} \right)^2 \cdot 12 = \left( \frac{2}{3} \right)^2 \cdot \left( \frac{3^2}{2^3} \right)^2 \cdot 2^2 \cdot 3 = 2^2 \cdot 3^4 \cdot 2^2 \cdot 3 = \frac{2^4 \cdot 3^5}{3^2 \cdot 2^6} = \frac{3^3}{2^2}
   \]
4) There were twelve bananas on the counter. One-sixth of the bananas were eaten yesterday. Three-fourths of the bananas were eaten today. What fraction of the bananas have been eaten in all?

The total is \( \frac{1}{6} + \frac{3}{4} = \frac{2}{12} + \frac{9}{12} = \frac{11}{12} \)

Answer: \( \frac{11}{12} \) of the bananas have been eaten

5) \( \frac{3}{5} \) of the rabbits on the farm are grey, \( \frac{2}{3} \) of the rest are white, and the remaining rabbits, 20, are black. How many rabbits are on the farm?

Answer: The total is \( 15 \times 10 = 150 \) rabbits

6) Martha bought a watch at 80% of the regular price. She paid €33.6 for the watch. What was the regular price?

\[
\frac{33.6}{80} = \frac{x}{100} \Rightarrow x = \frac{33.6 \cdot 100}{80} = 42
\]

Answer: The regular Price was €42

7) Tommy paid €21 for a book he purchased from a book club. This was a saving of €7 off the regular price. What percent discount did the book club give Tommy?

If Tommy paid €21, the regular price was €28, so

\[
\frac{7}{x} = \frac{28}{100} \Rightarrow x = \frac{7 \cdot 100}{28} = 25
\]

Answer: the percent of discount was 25%