EXAM 2_3

1. The graph below illustrates the temperature on a particular day as a function of time since midnight. (1 point)

a. What was the temperature at 3:00 a.m.?

b. When was the temperature 5 degrees?

c. When was the temperature below freezing? (less than 0 degrees)

d. When was the temperature increasing?

2. Find the equations of the following lines. (2 points)

3. Calculate the sum to 40 terms of an arithmetic progression whose first and eighth terms are 5 and 26. (1.5 points)

4. Work out the equations of the following lines and sketch them: (3 points)

   a) The line joining these points: \( A\left(-1, \frac{3}{4}\right) \) and \( B\left(2, -\frac{1}{4}\right) \)

   b) The line passes through \((2, -2)\) and a slope of \(\frac{1}{2}\).

   c) The line passes through \((1, -2)\) and cuts the y-axis in 3.

5. Solve by graphing and using another method (substitution, elimination or equalization):

   a) \[
   \begin{align*}
   3 + 4x &= 2y \\
   2x - y &= -1
   \end{align*}
   \]

   b) \[
   \begin{align*}
   \frac{x + 3y}{2} &= 5 \\
   3x - y &= 5y
   \end{align*}
   \] (2.5 points)
SOLUTION

1. The graph below illustrates the temperature on a particular day as a function of time since midnight.
   a. What was the temperature at 3:00 a.m.? 5 degrees below zero (-5º)
   b. When was the temperature 5 degrees? At 13:30 and 19:00 aprox.
   c. When was the temperature below freezing? (less than 0 degrees) between 1 and 10:30 aprox.
   d. When was the temperature increasing? From 6 to 16.

2. Find the equations of the following lines.
   a) $y = 4$ (horizontal line)
   b) $y = \frac{1}{2}x - 2$
   c) $y = -2x - 1$
   d) $y = \frac{5}{4}x + 5$

3. Calculate the sum to 40 terms of an arithmetic progression whose first and eighth terms are 5 and 26.

   $AP \rightarrow a_1 = 5, \quad a_8 = 26 \rightarrow a_8 = a_1 + 7d \rightarrow 26 = 5 + 7d \Rightarrow d = 3$

   $a_{40} = a_1 + 39d = 5 + 39 \cdot 3 = 122 \rightarrow S_{40} = \frac{(5+122) \cdot 40}{2} = 2540$

   Answer: the sum of the first 40 terms is 2540
4. Work out the equations of the following lines and sketch them:

a) The line joining these points: \( A\left(-\frac{3}{4}\right) \) and \( B\left(2, -\frac{1}{4}\right) \)

Slope: \( m = \frac{-\frac{3}{4} - \frac{1}{4}}{2 + 1} = -\frac{1}{3} \)

Line: \( y = 3 \cdot \frac{1}{3} (x + 1) \rightarrow y = -\frac{1}{3} x + \frac{5}{12} \)

b) The line passes through \((2,-2)\) and a slope of \(\frac{1}{2}\).

\( y = -2 + \frac{1}{2} (x - 2) \rightarrow y = -2 + \frac{1}{2} x - 1 \rightarrow y = \frac{1}{2} x - 3 \)

c) The line passes through \((1,-2)\) and cuts the y-axis in \( (0,3) \).

Slope: \( m = \frac{3 + 2}{0 - 1} = -5 \)

Line: \( y = 3 - 5(x - 0) \rightarrow y = -5x + 3 \)

5. Solve by graphing and using another method:

a) \( \begin{cases} 3 + 4x = 2y \\ 2x - y = -1 \end{cases} \) \rightarrow \begin{cases} y = \frac{3 + 4x}{2} \\ y = 2x + 1 \end{cases} \)

\( \frac{3 + 4x}{2} = 2x + 1 \rightarrow 3 + 4x = 4x + 2 \rightarrow 0x = 1 \) NO

solution by graphing: parallel lines, no solution
b) \[
\begin{align*}
\frac{x + 3y}{2} &= 5 \\
3x - y &= 5y
\end{align*}
\]
\[\Rightarrow \begin{align*}
x + 3y &= 10 \\
3x - 6y &= 0
\end{align*}\]
\[
\Rightarrow \begin{align*}
y &= \frac{10 - x}{3} \\
y &= \frac{1}{2}x
\end{align*}
\]
\[
\begin{align*}
x &= \frac{10 - x}{2} \\
3x + 2x &= 20\Rightarrow 5x &= 20 \\
x &= 4\Rightarrow y &= \frac{x}{2} = \frac{4}{2} = 2
\end{align*}
\]