EXAM UNIT 12 (STATISTICS)

1. Find two examples of each: (2 points)
   a) non ordered qualitative characters
   b) ordered qualitative characters
   c) discrete quantitative character
   d) continuous quantitative character

2. Following are the data represent the lives of a model of refrigerators: (5 p)

<table>
<thead>
<tr>
<th>Life (years)</th>
<th>[0,4)</th>
<th>[4,8)</th>
<th>[8,12)</th>
<th>[12,16)</th>
<th>[16,20)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refrigerators</td>
<td>2</td>
<td>7</td>
<td>16</td>
<td>13</td>
<td>2</td>
</tr>
</tbody>
</table>

a) Prepare a frequency table.

b) Find the modal and median classes.

c) Find the mean and the range.

d) Find the variance and standard deviation.

e) Build a histogram and the frequency polygon to represent the data.

f) What is the percentage of refrigerators that last less than 8 years?

3. Solve the following equations: (2 p)
   a) \((x+4)^2 - (2x - 1)^2 = 24x\)
   b) \(\frac{3x+2}{5} - \frac{4x-1}{10} + \frac{5x-2}{8} = \frac{x+1}{4}\)

4. Solve the following system by two different methods: graphing and addition or substitution.
\[
\begin{align*}
  x + y &= 0 \\
  \frac{x+3}{4} + 2y &= -1
\end{align*}
\]
SOLUTION

1. Find two examples of each:
   a) non ordered qualitative characters: Place of birth, professions.
   b) ordered qualitative characters: Months of birth,
   c) discrete quantitative character: Number of brothers and sisters, number of rooms in a group of houses.
   d) continuous quantitative character: weight and age of people from Seville.

2. Following are the data represent the lives of a model of refrigerators:

<table>
<thead>
<tr>
<th>Life (years)</th>
<th>Refrigerators</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a) Prepare a frequency table.

<table>
<thead>
<tr>
<th>Interval</th>
<th>( x_i )</th>
<th>( f_i )</th>
<th>( F_i )</th>
<th>( x_i \cdot f_i )</th>
<th>( x_i^2 \cdot f_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0,4)</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>[4,8)</td>
<td>6</td>
<td>7</td>
<td>9</td>
<td>42</td>
<td>252</td>
</tr>
<tr>
<td>[8,12)</td>
<td>10</td>
<td>16</td>
<td>25</td>
<td>160</td>
<td>1600</td>
</tr>
<tr>
<td>[12,16)</td>
<td>14</td>
<td>13</td>
<td>38</td>
<td>182</td>
<td>2548</td>
</tr>
<tr>
<td>[16,20)</td>
<td>18</td>
<td>40</td>
<td></td>
<td>36</td>
<td>648</td>
</tr>
<tr>
<td></td>
<td>( N = 40 )</td>
<td></td>
<td></td>
<td>424</td>
<td>5056</td>
</tr>
</tbody>
</table>

b) Find the modal and median classes. \( M_o = [8,12) \); \( M_e = [8,12) \)

c) Find the mean and the range. \( r = 20 - 0 = 20 \); \( \bar{x} = \frac{424}{40} = 10.6 \) years

d) Find the variance and standard deviation.

\[
\sigma^2 = \frac{\sum x_i^2 f_i}{N} - \overline{x}^2 = \frac{5056}{40} - 10.6^2 = 14.04 \rightarrow \sigma = 3.75 \text{ years}
\]
e) Build a histogram and the frequency polygon to represent the data.

![Histogram and Frequency Polygon]

f) What is the percentage of refrigerators that last less than 8 years?

Less than 8 years: 2 + 7 = 9

\[
\frac{x}{100} = \frac{7}{40} \rightarrow x = \frac{700}{40} = 17.5 \% \text{ of the fridges last less than 8 years}
\]

3. Solve the following equations:

a) \((x + 4)^2 - (2x - 1)^2 = 24x \rightarrow x^2 + 8x + 16 - 4x^2 - 4x - 1 = 24x\)

\[-3x^2 - 12x + 15 = 0 \rightarrow x = \frac{12 \pm \sqrt{144 + 180}}{-6} = \frac{12 \pm 18}{-6} = \frac{-5}{1}\]

b) \(\frac{3x + 2}{5} - \frac{4x - 1}{10} + \frac{5x - 2}{8} = \frac{x + 1}{4} \rightarrow 24x + 16 - 16x + 4 + 25x - 10 = 10x + 10 \rightarrow 23x = 0 \rightarrow x = 0\)
4. Solve the following system by two different methods: graphing and addition or substitution.

\[
\begin{align*}
  x + y &= 0 \\
  \frac{x + 3}{4} + 2y &= -1 \quad \rightarrow \quad y = -x \\
  x + 3 + 8y &= -4 \quad \rightarrow \quad y = -x - 7
\end{align*}
\]

Substitution:

\[
\begin{align*}
  y &= -x \\
  x + 3 + 8y &= -4
\end{align*}
\]

\[
\begin{align*}
  -7x &= -7 \Rightarrow x = 1 \\
  y &= -x \quad \Rightarrow \quad y = -1
\end{align*}
\]

Solution:

\[
\begin{align*}
  x &= 1 \\
  y &= -1
\end{align*}
\]