EXAM 3_3  (Coordinate Geometry_Functions)

1. A circle has center at (0,4) and passes through the point (3,0). Find an equation to this circle. (1.5 points)

2. A triangle has vertices A(-3,3), B(-3,-2), C(5,-2). Show that it is a right angled triangle. Calculate its perimeter and area. (1.5 points)

3. Find the equation of the line which passes through the origin and the point of intersection of \( y = x + 4 \) and \( x + y = 6 \). (1.75 points)

4. Find the equation of the perpendicular bisector of the line segment AB, with A(-1,2) and B(1,4). Find a parallel line to AB, passing through the point P(-3,2). (1.75 points)

5. A quadrilateral has vertices A(5,4), B(-2,5), C(-1,-2) and D(6,-3). Show that the quadrilateral is a rhombus and calculate the area of ABCD. (1.75 points)
6. Match the equations to the corresponding graphs (explaining your answer):

a) \( y = -2x^2 + 4x \)  

b) \( y = \frac{4}{x+1} \)  

c) \( y = 2^{x-1} \)  

d) \( y = -x^2 + 2x + 1 \)  

e) \( y = \log_{\frac{1}{2}} x \)  

f) \( y = -\frac{4}{x-1} \)
SOLUTION

1. A circle has center at (0,4) and passes through the point (3,0). Find an equation to this circle

\[(x-0)^2 + (y-4)^2 = r^2\]
It passes through the point (3,0)
\[3^2 + (0-4)^2 = r^2\]
\[9 + 16 = r^2 \rightarrow r^2 = 25 \rightarrow \text{Equation: } x^2 + (y-4)^2 = 25\]

2. A triangle has vertices A(-3,3), B(-3,-2), C(5,-2). Show that it is a right angled triangle. Calculate its perimeter and area.

If it was a right triangle \(\rightarrow\) Pythagorean Theorem:
\[b^2 + c^2 = h^2\]
We are going calculate the distances, to get the legs and hypotenuse:
\[d(A,B) = \sqrt{(-3+3)^2 + (-2-3)^2} = \sqrt{0+25} = 5\,u\]
\[d(A,C) = \sqrt{(5+3)^2 + (-2-3)^2} = \sqrt{64+25} = \sqrt{89}\,u \rightarrow \text{hypotenuse? the biggest}\]
\[d(B,C) = \sqrt{(5+3)^2 + (-2+2)^2} = \sqrt{64+0} = 8\,u\]
\[\overline{AC}^2 = \overline{AB}^2 + \overline{BC}^2 \rightarrow \left(\sqrt{89}\right)^2 = 5^2 + 8^2 \rightarrow 89 = 25 + 64 \rightarrow 89 = 89\]
So, yes, it is a right angled triangle.
Perimeter: \[P = 5 + 8 + \sqrt{89} = 13 + \sqrt{89} \,u\]
Area: \[A = \frac{5 \times 8}{2} = 20\,u^2\]

3. Find the equation of the line which passes through the origin and the point of intersection of \(y = x + 4\) and \(x + y = 6\).

Point A (0,0), point B:
\[y = x + 4\]
\[x + y = 6 \rightarrow x + x + 4 = 6 \rightarrow 2x = 2 \rightarrow x = 1\]
\[y = 1 + 4 = 5 \rightarrow B(1,5)\]

Equation \(\overline{AB} : \frac{x-0}{1-0} = \frac{y-0}{5-0} \rightarrow 5x = y \rightarrow y = 5x\)
4. Find the equation of the perpendicular bisector of the line segment AB, with A(-1,2) and B(1,4). Find a parallel line to AB, passing through the point P(-3,2).

Equation of $AB: \frac{x+1}{1+1} = \frac{y-2}{4-2} \Rightarrow 2(x+1) = 2(y-2) \Rightarrow x + 1 = y + 2 \Rightarrow y = x - 1$

Perpendicular bisector (It passes through the midpoint of AB, perpendicular):

$M_{AB} = \left( \frac{-1+1}{2}, \frac{2+4}{2} \right) = (0,3) \hline \text{slope:} \hline m' = -\frac{1}{m} = -\frac{1}{1} = -1 \hline y - 3 = -1(x - 0) \Rightarrow y = -x + 3$ Perpendicular bisector

Parallel to $AB$ passing through (-3,2): slope $m = 1$

$y - 2 = 1(x + 3) \Rightarrow y = x + 5$ parallel line to $AB$

5. A quadrilateral has vertices A(5,4), B(-2,5), C(-1,-2) and D(6,-3). Show that the quadrilateral is a rhombus and calculate the area of ABCD.

If the quadrilateral is a rhombus, its diagonals are perpendicular lines. We are going to find the equations of the diagonals AC and BD:

\[ \overline{AC} : \frac{x - 5}{-1-5} = \frac{y - 4}{-2-4} \Rightarrow -6(x - 5) = -6(y - 4) \Rightarrow x = y - 4 \Rightarrow y = x - 1 \]

\[ \overline{BD} : \frac{x + 2}{6+2} = \frac{y - 5}{-3-5} \Rightarrow -8(x + 2) = 8(y - 5) \Rightarrow -x - 2 = y - 5 \Rightarrow y = -x + 3 \]

We study the slopes: $m_{AC} = 1, \quad m_{BD} = -1 \Rightarrow$ perpendicular, so it is a rhombus.

Area: $A = \frac{D \times d}{2}$

\[ d = d(A,C) = \sqrt{(-1-5)^2 + (-2-4)^2} = \sqrt{36 + 36} = \sqrt{72} = 6\sqrt{2} \ u \]

\[ D = d(B,D) = \sqrt{(6+2)^2 + (-3-5)^2} = \sqrt{64 + 64} = \sqrt{128} = 8\sqrt{2} \ u \]

\[ A = \frac{D \times d}{2} = \frac{8\sqrt{2} \times 6\sqrt{2}}{2} = 48 \ u^2 \]
6. Match the equations to the corresponding graphs (explaining your answer):

a) \( y = -2x^2 + 4x \) \ Parabola \cap \text{passing through (0,0)} \rightarrow \text{Graph VI}

b) \( y = \frac{4}{x + 1} \) \ Hyperbole, asymptote \( x = -1 \) \rightarrow \text{Graph III}

c) \( y = 2^{x-1} \) \ Exponential, asymptote \( x \)-axis, passing (0,1/2) \rightarrow \text{Graph I}

d) \( y = -x^2 + 2x + 1 \) \ Parabola \cap \text{passing through (0,1)} \rightarrow \text{Graph II}

e) \( y = \log_2 x \) \ Logarithmic, asymptote \( y \)-axis, passing (1,0) \rightarrow \text{Graph V}

f) \( y = -\frac{4}{x - 1} \) \ Hyperbole, asymptote \( x = 1 \) \rightarrow \text{Graph IV}