EXAM 1_3 (Algebra - Equations)

1. Solve the following: (1.5 points)

$$\left.\begin{array}{l} y^2 - 2y + 1 = x \\ \sqrt{x} + y = 5 \end{array}\right\}$$

2. Solve the following: (4 points)

a) $\dfrac{1}{x^2 - x} - \dfrac{1}{x - 1} = 0$

b) $x - \sqrt{3x - 5} = 3$

c) $2x^3 + 5x^2 + x - 2 = 0$

d) $\dfrac{2}{x^2 - 9} = \dfrac{x^2 - 16}{72}$

3. Mr. B. has $20,000 to invest. He invests part at 6%, the rest at 7%, and he earns $1,280 interest. How much did he invest at each rate?   (1.5 p)

4. The Lakers scored a total of 80 points in a basketball game against the Bulls. The Lakers made a total of 37 two-points and three-points baskets. How many three-points shots did the Lakers make? How many two-points shots did the Lakers make?   (1.5 p)

5. The length of a rectangle is 7 metres less than twice the width. Find the dimensions if the area is 60 square metres.   (1.5 p)

SOLUTION

1. $\left.\begin{array}{l} y^2 - 2y + 1 = x \\ \sqrt{x} + y = 5 \end{array}\right\} \rightarrow y = 5 - \sqrt{x} \rightarrow (5 - \sqrt{x})^2 - 2(5 - \sqrt{x}) + 1 = x$

$25 - 2 \cdot 5\sqrt{x} + x - 10 + 2\sqrt{x} + 1 = x \rightarrow 25 - 10\sqrt{x} + 2\sqrt{x} - 10 + 1 = 0$

$16 = 8\sqrt{x} \rightarrow 2 = \sqrt{x} \rightarrow x = 4 \Rightarrow y = 5 - \sqrt{x} = 5 - 2 = 3$ Solution: x = 4, y = 3

2. Solve the following:

a) $\dfrac{1}{x^2 - x} - \dfrac{1}{x - 1} = 0 \rightarrow \dfrac{1}{x(x-1)} - \dfrac{x}{x(x-1)} = \dfrac{0}{x(x-1)} \rightarrow 1 - x = 0 \Rightarrow x = 1$

But x = 1 is not a solution. Denominator cancels $\dfrac{1}{0} - \dfrac{1}{-1} = 0$

So, it doesn't have any solution

b) $x - \sqrt{3x - 5} = 3 \rightarrow x - 3 = \sqrt{3x - 5} \rightarrow (x - 3)^2 = \left(\sqrt{3x - 5}\right)^2$

$x^2 - 6x + 9 = 3x - 5 \Rightarrow x^2 - 9x + 14 = 0 \rightarrow x = \dfrac{9 \pm \sqrt{81 - 56}}{2} = \Big\langle \begin{array}{l} 7 \\ 2 \end{array}$

Checking: $\Big\langle \begin{array}{l} 7 - \sqrt{21 - 5} = 3 \rightarrow 7 - 4 = 3 \text{ YES} \\ 2 - \sqrt{6 - 5} = 3 \rightarrow 2 - 2 \neq 3 \quad \text{NO} \end{array}$ Solution x = 7

c) $2x^3 + 5x^2 + x - 2 = 0$ Factors(2) = 1, -1, 2, -2

$P(1) = 2 + 5x^2 + 1 - 2 \neq 0$; $P-(1) = -2 + 5x^2 - 1 - 2 = 0$

Ruffini

	2	+5	+1	-2
-1		-2	-3	+2
	2	+3	-2	0

$2x^2 + 3x - 2 = 0$

$x = \dfrac{-3 \pm \sqrt{9 + 16}}{4} = \Big\langle \begin{array}{l} \frac{1}{2} \\ -2 \end{array}$

$2x^3 + 5x^2 + x - 2 = 0 \rightarrow 2(x + 1)(x + 2)\left(x - \dfrac{1}{2}\right) = 0 \Rightarrow$ Solution : $\begin{cases} x_1 = -1 \\ x_2 = -2 \\ x_3 = \dfrac{1}{2} \end{cases}$

d) $\dfrac{2}{x^2 - 9} = \dfrac{x^2 - 16}{72} \rightarrow 144 = (x^2 - 16)(x^2 - 9) \rightarrow 144 = x^4 - 16x^2 - 9x^2 + 144$

$x^4 - 25x^2 = 0 \Rightarrow x^2(x^2 - 25) = 0 \Rightarrow \Big\langle \begin{array}{l} x^2 = 0 \rightarrow x = 0 \\ x^2 - 25 = 0 \rightarrow x = \pm 5 \end{array}$

3. Mr. B. has $20,000 to invest. He invests part at 6%, the rest at 7%, and he earns $1,280 interest. How much did he invest at each rate?

Investment at 6% - x

$x + y = 20000$

Investment at 7% - y

$\dfrac{6}{100}x + \dfrac{7}{100}y = 1280$

$\left.\begin{array}{l} x + y = 20000 \\ 0.06x + 0.07y = 1280 \end{array}\right\} \rightarrow y = 20000 - x \Rightarrow 0.06x + 0.07(20000 - x) = 1280$

$0.06x + 1400 - 0.07x = 1280 \Rightarrow -0.01x = -120 \Rightarrow x = 12000$

He invested $12000 at 6% and $8000 at 7%

4. The Lakers scored a total of 80 points in a basketball game against the Bulls. The Lakers made a total of 37 two-points and three-points baskets. How many three-points shots did the Lakers make? How many two-points shots did the Lakers make?

Two-points shots – x

$x + y = 37$

Three-points shots – y

$2x + 3y = 80$

$\left.\begin{array}{l} x + y = 37 \\ 2x + 3y = 80 \end{array}\right\} \rightarrow \left.\begin{array}{l} -2x - 2y = -74 \\ 2x + 3y = 80 \end{array}\right\} \Rightarrow y = 6 \rightarrow x = 37 - 6 = 31$

Answer: They made 6 three-points shots and 31 two-points shots

5. The length of a rectangle is 7 metres less than twice the width. Find the dimensions if the area is 60 square metres.

Width x Length y

$\left.\begin{array}{l} y = 2x - 7 \\ x \cdot y = 60 \end{array}\right\} \Rightarrow x \cdot (2x - 7) = 60 \rightarrow 2x^2 - 7x - 60 = 0 \rightarrow x = \dfrac{7 \pm \sqrt{49 + 480}}{4} = \left\langle\begin{array}{l} \dfrac{15}{2} \\ -4 \end{array}\right.$

So $x = \dfrac{15}{2} \Rightarrow y = 2 \cdot \dfrac{15}{2} - 7 = 8$

Answer: lenght 8 metres and width 7.5 metres