Maths 4^{th} ESO

EXAM 2_1 (FUNCTIONS/INEQUALITIES)

Name:.____

1. Plot the function: $f(x)=\begin{cases} 2-x & \text{if } x<-1 \\ 3 & \text{if } -1<x\le 3 \\ 2x-5 & \text{if } x>3 \end{cases}$ (2p)

And find:

a) Its domain and range.
b) Continuity.
c) Increasing and decreasing intervals.

2. Find the domain of the following functions: (2.25 p)

$f(x)=\dfrac{x^3+3}{x^2-9x+8}$; $g(x)=\sqrt[3]{\dfrac{x}{x^2-1}}$; $h(x)=\sqrt{\dfrac{x+1}{9-x^2}}$

3. Given the equation of the parabola $f(x)=-x^2+4x-3$ (1.5 p)

a) Find its vertex and symmetry axis.
b) Its intersections with the x axis and the y axis.
c) Draw the graph of f(x).
d) Find the range of f(x).

4. Solve by graphing the simultaneous equation: $\left.\begin{array}{l} y=x^2-2x \\ y=x+4 \end{array}\right\}$ (2 p)

(Write the steps you have taken to reach the solution)

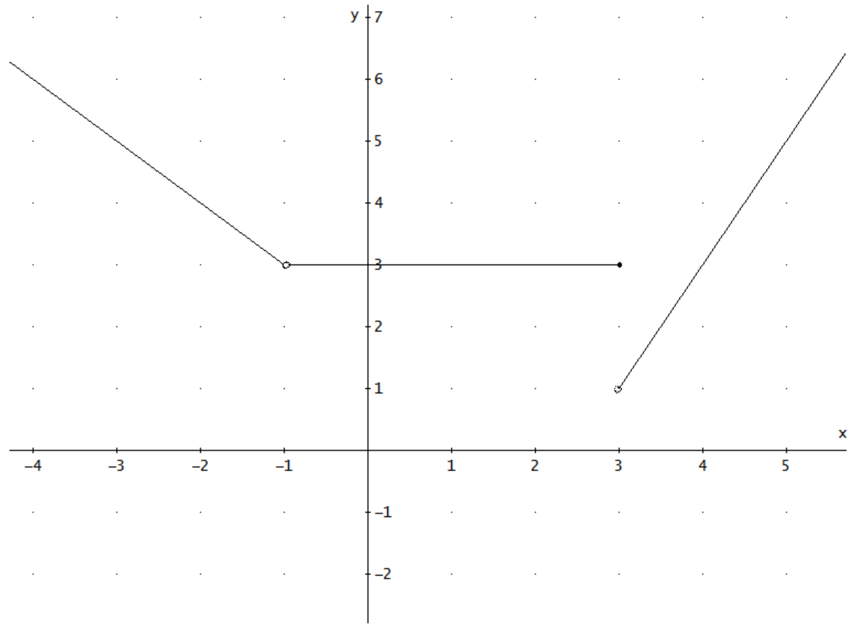
5. Solve the following systems of inequalities: (2.25 p)

a) $\left.\begin{array}{l} x+2y\le 10 \\ y>x-2 \end{array}\right\}$

b) $\left.\begin{array}{l} 2(x+1)>x-7 \\ \dfrac{2x-3}{3}\le 2(x+1)-1 \\ 2x-5<x-5 \end{array}\right\}$

SOLUTION

2. Plot the function: $f(x)=\begin{cases}2-x & \text{if } x<-1\\ 3 & \text{if } -1<x\le 3\\ 2x-5 & \text{if } x>3\end{cases}$



a) Its domain and range. $Dom(f)=\Re-\{-1\}$; $Range(f)=(1,+\infty)$

b) Continuity. It is continuous in $(-\infty,-1)\cup(-1,3)\cup(3,+\infty)$

It has a jump discontinuity in x = 3 and a removable discontinuity in x =-1.

c) Increasing and decreasing intervals: Decreasing in $(-\infty,-1)$, Increasing in $(3,+\infty)$ and constant in $(-1,3)$

2. Find the domain of the following functions:

$f(x)=\frac{x^3+3}{x^2-9x+8}\rightarrow x^2-9x+8=0\rightarrow x=\frac{9\pm\sqrt{49}}{2}=\langle\begin{matrix}8\\1\end{matrix}$; $Dom(f)=\Re-\{1,8\}$

$g(x)=\sqrt[3]{\frac{x}{x^2-1}}\rightarrow x^2-1=0\rightarrow x=\pm1$; $Dom(g)=\Re-\{1,-1\}$

$h(x)=\sqrt{\frac{x+1}{9-x^2}}\rightarrow\frac{x+1}{9-x^2}\ge0\rightarrow x=-1; x=3; x=-3$, we study the sign by intervals:

In $(-\infty,-3)\rightarrow+$; in $(-3,-1)\rightarrow-$; in $(-1,3)\rightarrow+$; in $(3,+\infty)\rightarrow-$

So, the Domain is $Dom(h)=(-\infty,-3)\cup[-1,3)$

3. Given the equation of the parabola $f(x) = -x^2 + 4x - 3 \rightarrow \cap$

Find its vertex and its symmetry axis

$x = -\frac{4}{-2} = 2 \rightarrow$ Vertex (2, 1), symmetry axis: x = 2

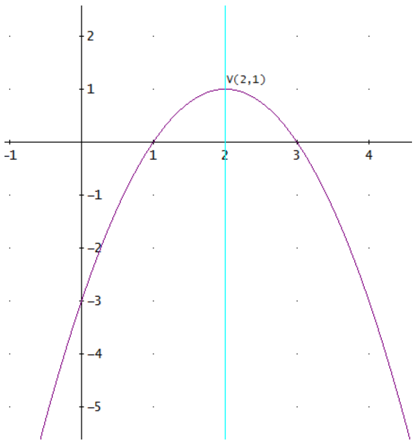
Intersections with the x-axis and the y-axis. Y-axis (0,-3)

X axis $-x^2 + 4x - 3 = 0 \rightarrow x = \frac{-4 \pm \sqrt{4}}{-2} = \left\langle \begin{matrix} 1 \\ 3 \end{matrix} \right.$

Draw the graph of f(x).

Find the range of f(x).

$\text{Range}(f) = (-\infty, 1]$

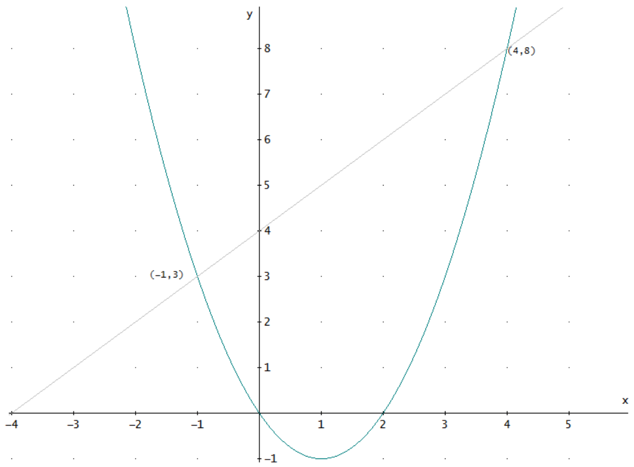


4. Solve by graphing the simultaneous equation: $\left. \begin{matrix} y = x^2 - 2x \\ y = x + 4 \end{matrix} \right\}$

(Write the steps you have taken to reach the solution)

we draw the parabola and the line: $y = x^2 - 2x \rightarrow \cup$; vertex: $(1, -1)$

y-intercepts $\rightarrow (0,0)$; x-intercepts $\rightarrow x^2 - 2x = 0 \rightarrow x(x-2) = 0 \rightarrow \left\langle \begin{matrix} x = 0 \\ x = 2 \end{matrix} \right.$



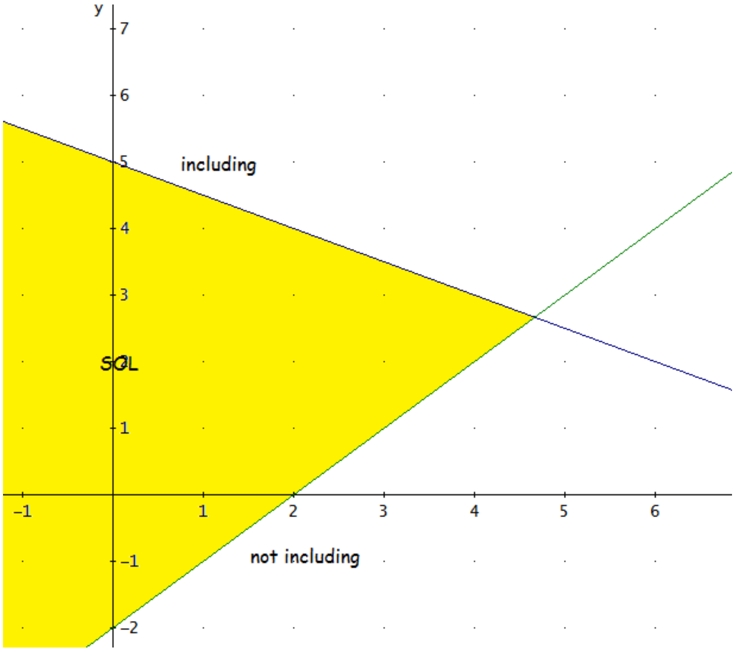
Solution:

x=-1, y=3

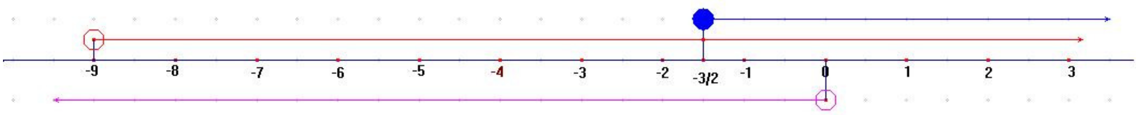
x= 4, y=8

5. Solve the following systems of inequalities:

a) $\left.\begin{array}{l} x+2y\le 10 \\ y>x-2 \end{array}\right\}$ by graphing $\left.\begin{array}{l} x+2y=10 \\ y=x-2 \end{array}\right\} \rightarrow \left.\begin{array}{l} y=\dfrac{10-x}{2} \\ y=x-2 \end{array}\right\}$



b) $\left.\begin{array}{l} 2(x+1)>x-7 \\ \dfrac{2x-3}{3}\le 2(x+1)-1 \\ 2x-5<x-5 \end{array}\right\} \rightarrow \left.\begin{array}{l} 2x+2>x-7 \\ 2x-3\le 6x+6-3 \\ 2x-x<-5+5 \end{array}\right\} \rightarrow \left.\begin{array}{l} x>-9 \\ -4x\le 6 \\ x<0 \end{array}\right\} \rightarrow \left.\begin{array}{l} x>-9 \\ x\ge -\dfrac{3}{2} \\ x<0 \end{array}\right\}$



Solution: $\left[-\dfrac{3}{2},0\right)$