1st TERM GENERAL EXAM

Remember: in each question, write the steps you have taken to reach the solution. (1 point each question)

1) Classify according to number type and mark on the real number line the following. (Notice that some numbers may be of more than one type).

a. $-\sqrt{5}$ b. $-1.\hat{6}$

a. $\sqrt{16}$ d. $1.1\hat{6}$

2) Calculate and simplify:

a. $5\sqrt[6]{64a^2}-5\sqrt[3]{27a}+6\sqrt[9]{a^3}$ b. $\left(3\sqrt[4]{4a^2b^3}\cdot\sqrt{2ab}\right)^3$

3) Rationalize and simplify: $\dfrac{\sqrt{a}}{\sqrt{a}+\sqrt{b}}+\dfrac{\sqrt{b}}{\sqrt{a}-\sqrt{b}}$

4) Solve: $\dfrac{x^2(x-3)^2}{2}-18=x^2(7-3x)$

5) Solve: $x-\sqrt{169-x^2}=17$

6) Solve: $\dfrac{x+5}{x^2-4}-\dfrac{x-4}{x^2+4x+4}=0$

7) Solve the following simultaneous equation: $\left.\begin{array}{l}\dfrac{3}{x}+\dfrac{2}{y}=1\\ x\cdot y=24\end{array}\right\}$

8) Solve the following system of inequalities: $\left.\begin{array}{l}1-(2x-1)<0\\ \dfrac{x-1}{6}-\dfrac{x-3}{2}\le -1\\ x-3\ge 2(x-2)\end{array}\right\}$

9) Solve: $2x-(x-3)(x+3)\le 37-3(x-4)^2$

10) If the numerator and denominator of a fraction are both decreased by one the fraction becomes $\dfrac{2}{3}$. If the numerator and denominator are both increased by one the fraction becomes $\dfrac{3}{4}$. Find the original fraction.

SOLUTION

1) Classify according to number type and mark on the real number line the following. (Notice that some numbers may be of more than one type).

a. $-\sqrt{5}$ Irrational number, negative

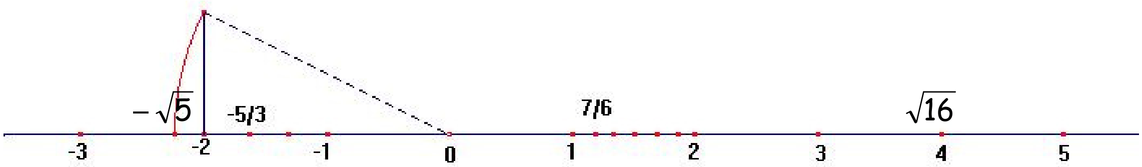
b. $-1.\widehat{6}$ Rational, recurring decimal, negative

$$N = 1.666\ldots \begin{cases} 10N = 16.666\ldots \\ N = 1.666\ldots. \end{cases} \rightarrow 9N = 15 \rightarrow N = \frac{15}{9} \rightarrow -1.\widehat{6} = -\frac{5}{3}$$

c. $\sqrt{16}$ = 4 Natural number

d. $1.1\widehat{6}$ Rational, recurring decimal, positive

$$N = 1.166666\ldots \begin{cases} 100N = 116.666\ldots \\ 10N = 11.666\ldots. \end{cases} \rightarrow 90N = 105 \rightarrow N = \frac{105}{90} = \frac{7}{6}$$



2) Calculate and simplify:

a. $5\sqrt[6]{64a^2} - 5\sqrt[3]{27a} + 6\sqrt[9]{a^3} = 5\sqrt[6]{2^6a^2} - 5\sqrt[3]{3^3a} + 6\sqrt[9]{a^3} = 10\sqrt[6]{a^2} - 15\sqrt[3]{a} + 6\sqrt[9]{a^3} =$

$= 10\sqrt[3]{a} - 15\sqrt[3]{a} + 6\sqrt[3]{a} = \sqrt[3]{a}$

b. $\left(3\sqrt[4]{4a^2b^3}\cdot\sqrt{2ab}\right)^3 = \left(3\sqrt[4]{2^2a^2b^3}\cdot\sqrt[4]{2^2a^2b^2}\right)^3 = \left(3\sqrt[4]{2^4a^4b^5}\right)^3 = 3^3\left(2ab\sqrt[4]{b}\right)^3 =$

$= 3^3\left(2^3a^3b^3\sqrt[4]{b^3}\right) = 6^3a^3b^3\sqrt[4]{b^3}$

3) $\dfrac{\sqrt{a}}{\sqrt{a}+\sqrt{b}} + \dfrac{\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \dfrac{\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)}{\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right)} + \dfrac{\sqrt{b}\left(\sqrt{a}+\sqrt{b}\right)}{\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right)} =$

$= \dfrac{a-\sqrt{ab}}{\left(\sqrt{a}\right)^2-\left(\sqrt{b}\right)^2} + \dfrac{\sqrt{ba}+b}{\left(\sqrt{a}\right)^2-\left(\sqrt{b}\right)^2} = \dfrac{a-\sqrt{ab}+\sqrt{ab}+b}{a-b} = \dfrac{a+b}{a-b}$

4) Solve: $\dfrac{x^2(x-3)^2}{2} - 18 = x^2(7-3x) \rightarrow \dfrac{x^2\left(x^2-6x+9\right)}{2} - \dfrac{36}{2} = \dfrac{2x^2(7-3x)}{2}$

$x^4 - 6x^3 + 9x^2 - 36 = 14x^2 - 6x^3 \rightarrow x^4 - 5x^2 - 36 = 0 \rightarrow z = x^2$

$$z^2 - 5z - 36 = 0 \rightarrow z = \frac{5 \pm \sqrt{25 + 4 \cdot 36}}{2} = \left\langle \begin{array}{l} 9 \Rightarrow x = \sqrt{9} = \pm 3 \\ -4 \Rightarrow x = \sqrt{-4} \text{ Not real} \end{array} \right.$$

Solution: x = 3 and x = -3

5) Solve: $x - \sqrt{169 - x^2} = 17 \rightarrow x - 17 = \sqrt{169 - x^2}$

$$(x-17)^2 = \left(\sqrt{169 - x^2}\right)^2 \rightarrow x^2 - 34x + 289 = 169 - x^2 \rightarrow 2x^2 - 34x + 120 = 0$$

$$x^2 - 17x + 60 = 0 \rightarrow x = \frac{17 \pm \sqrt{289 - 240}}{2} = \frac{17 \pm 7}{2} = \left\langle \begin{array}{l} 12 \\ 5 \end{array} \right.$$

Checking: $x - \sqrt{169 - x^2} = 17 \rightarrow \left\langle \begin{array}{l} 12 - \sqrt{169 - 144} = 17 \rightarrow 12 - 5 \neq 17 \text{ NO} \\ 5 - \sqrt{169 - 25} = 17 \rightarrow 5 - 12 \neq 17 \text{ NO} \end{array} \right.$

It has not solution

6) Solve: $\frac{x+5}{x^2 - 4} - \frac{x-4}{x^2 + 4x + 4} = 0 \rightarrow \frac{x+5}{(x-2)(x+2)} - \frac{x-4}{(x+2)^2} = 0$

$$LCD = (x-2)(x+2)^2 \rightarrow \frac{(x+5)(x+2)}{(x-2)(x+2)^2} - \frac{(x-4)(x-2)}{(x-2)(x+2)^2} = 0$$

$x^2 + 7x + 10 - \left(x^2 - 6x + 8\right) = 0 \rightarrow 13x + 2 = 0 \rightarrow x = -\frac{2}{13}$ Solution

7) Solve the following simultaneous equation: $\left. \begin{array}{l} \frac{3}{x} + \frac{2}{y} = 1 \\ x \cdot y = 24 \end{array} \right\}$

$$\left. \begin{array}{l} \frac{3}{x} + \frac{2}{y} = 1 \\ x \cdot y = 24 \end{array} \right\} \rightarrow \left. \begin{array}{l} 3y + 2x = xy \\ y = \frac{24}{x} \end{array} \right\} \rightarrow 3 \cdot \frac{24}{x} + 2x = x \cdot \frac{24}{x} \rightarrow \frac{72}{x} + \frac{2x^2}{x} = \frac{24x}{x}$$

$$2x^2 - 24x + 72 = 0 \rightarrow x^2 - 12x + 36 = 0 \rightarrow (x-6)^2 = 0 \rightarrow x - 6 = 0 \rightarrow x = 6$$

$y = \frac{24}{x} = \frac{24}{6} = 4 \rightarrow$ Solution: x = 6 y = 4

8) Solve the following system of inequalities: $\left. \begin{array}{l} 1 - (2x-1) < 0 \\ \frac{x-1}{6} - \frac{x-3}{2} \leq -1 \\ x - 3 \geq 2(x-2) \end{array} \right\}$

$$\left. \begin{array}{l} 1 - (2x-1) < 0 \\ \frac{x-1}{6} - \frac{x-3}{2} \leq -1 \\ x - 3 \geq 2(x-2) \end{array} \right\} \rightarrow \left. \begin{array}{l} 1 - 2x + 1 < 0 \\ \frac{x-1}{6} - \frac{3x-9}{6} \leq -\frac{6}{6} \\ x - 3 \geq 2x - 4 \end{array} \right\} \rightarrow \left. \begin{array}{l} -2x < -2 \\ x - 1 - 3x + 9 \leq -6 \\ x - 2x \geq -4 + 3 \end{array} \right\}$$

$$\left.\begin{array}{l} 2x > 2 \\ -2x \le -6+1-9 \\ -x \ge -1 \end{array}\right\} \rightarrow \left.\begin{array}{l} x > \frac{2}{2} \\ 2x \ge 14 \\ x \le 1 \end{array}\right\} \rightarrow \left.\begin{array}{l} x > 1 \\ x \ge 7 \\ x \le 1 \end{array}\right\}$$

Solution: ϕ

9) Solve: $2x-(x-3)(x+3) \le 37-3(x-4)^2 \rightarrow 2x-\left(x^2-9\right) \le 37-3\left(x^2-8x+16\right)$

$2x-x^2+9 \le 37-3x^2+24x-48 \rightarrow 2x^2-22x+20 \le 0 \rightarrow x^2-11x+10 \le 0$

$x^2-11x+10=0 \rightarrow x=\dfrac{11 \pm \sqrt{121-40}}{2} = \left\langle \begin{array}{l} 10 \\ 1 \end{array} \right. \rightarrow (x-1)(x-10) \le 0$

	$-\infty$ to 1	1 to 10	10 to ∞
$(x-1)$	-	+	+
$(x-10)$	-	-	+
$(x-1)(x-10)$	+	-	+

Sol: $[1,10]$

10) If the numerator and denominator of a fraction are both decreased by one the fraction becomes $\frac{2}{3}$. If the numerator and denominator are both increased by one the fraction becomes $\frac{3}{4}$. Find the original fraction.

x numerator , y denominator

$$\left.\begin{array}{l} \frac{x-1}{y-1}=\frac{2}{3} \\ \frac{x+1}{y+1}=\frac{3}{4} \end{array}\right\} \rightarrow \left.\begin{array}{l} 3x-3=2y-2 \\ 4x+4=3y+3 \end{array}\right\} \rightarrow \left.\begin{array}{l} 3x-2y=1 \\ 4x-3y=-1 \end{array}\right\} \rightarrow x=\frac{1+2y}{3} \rightarrow 4\left(\frac{1+2y}{3}\right)-3y=-1$$

$$\frac{4+8y}{3}-3y=-1 \rightarrow 4+8y-9y=-3 \rightarrow -y=-7 \rightarrow y=7$$

$$x=\frac{1+2y}{3}=\frac{1+14}{3}=5$$

The original fraction was $\frac{5}{7}$