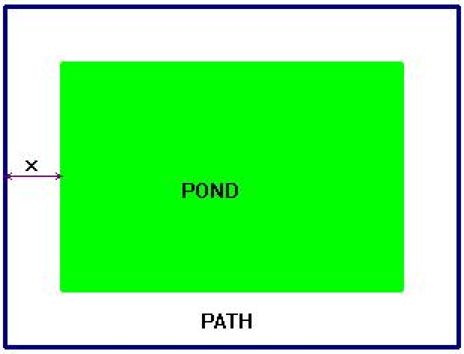
2nd TERM GENERAL EXAM

Remember: in each question, write the steps you have taken to reach the solution. (1 point each question)



1. A rectangular pond (6m x 4m), is surrounded by a uniform path of width x metres. The area of the path is equal to the area of the pond. Find x.

2. Solve the equation: $x+\sqrt{3x+10}=6$

3. Solve: $\dfrac{x+5}{x+3}=1-\dfrac{x^2+3x+6}{x^2+2x-3}$

4. Solve by substitution and graphically : $\left.\begin{array}{l} 2x-y-1=0 \\ y=-\dfrac{2}{x+1} \end{array}\right\}$

5. Solve the inequality: $\dfrac{(x-2)(x+2)}{4}-\dfrac{x-6}{2}\leq\dfrac{10+x}{5}$

6. Solve the system of inequalities: $\left.\begin{array}{l} 2x+\dfrac{3y}{4}\leq 5 \\ 10x-y>6 \end{array}\right\}$

7. Sketch the graph of the compound function:

$$f(x)=\begin{cases} 1-x & x<-3 \\ 4 & -3<x<2 \\ 1+\log_2 x & x\geq 2 \end{cases}$$

a) Domain and range

b) Continuity

8. Calculate x in the following equations:

a) $3^{x-2}=\dfrac{1}{27}$ b) $\log_{(2x+1)}49=2$

9. A ball is thrown in the air so that t seconds after it is thrown, its height h metres above it starting point is given by the function $h(t) = 25t - 5t^2$.

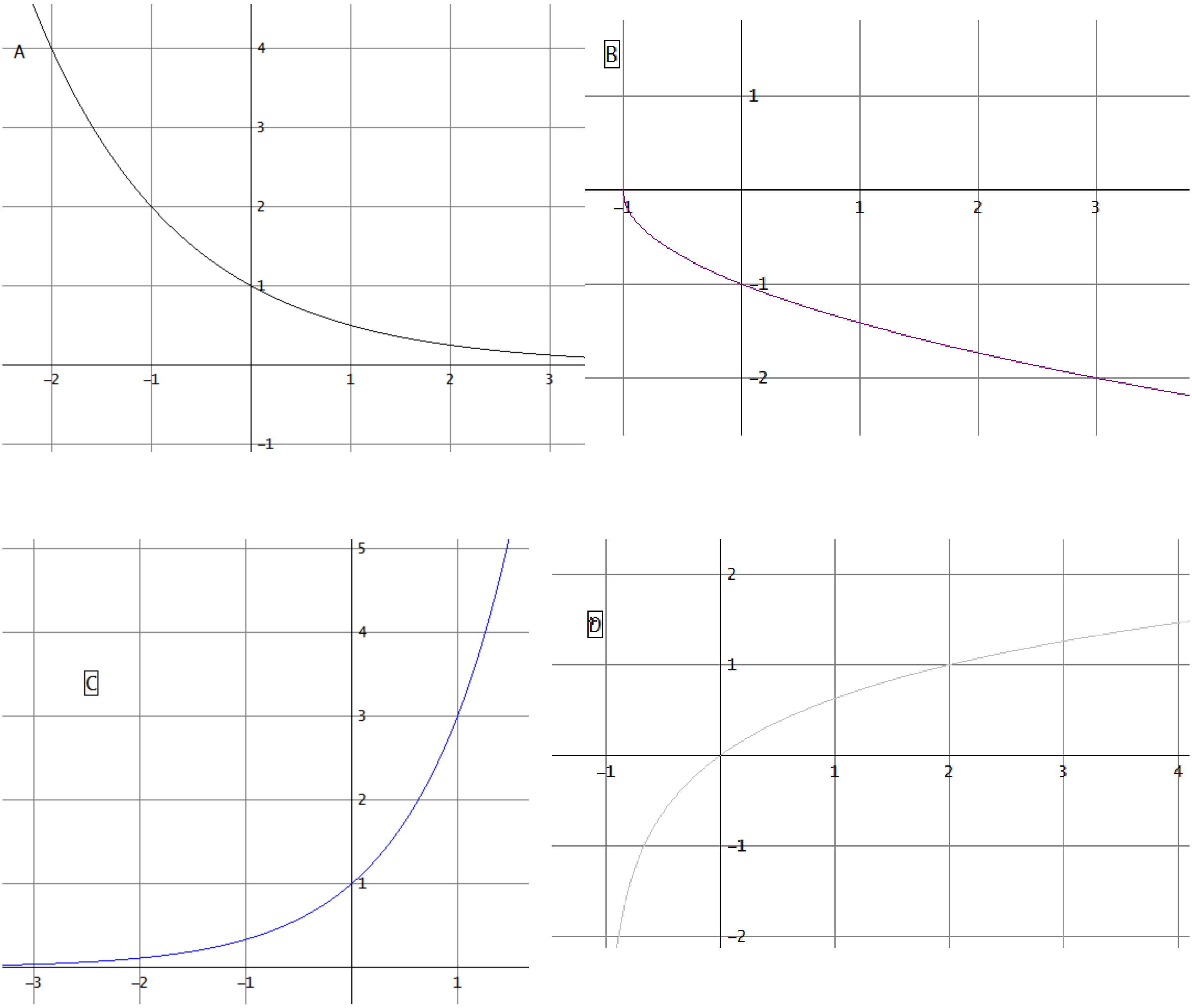
Draw the graph of the function of $0 \le t \le 6$ and find:

a) The time when the ball is at its greatest height.

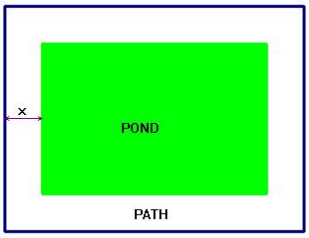
b) The greatest height reached by the ball.

c) The interval of time during which the ball is at a height of more than 30 m.

10. The diagrams show the graphs of four functions. Write their formulas (explaining why)



SOLUTION



1. Area of the pond $A = 6 \cdot 4 = 24 m^2$

Area of the path $24 m^2$

Area of the path+pond $48 m^2$

$A = (6 + 2x)(4 + 2x) = 48$

$24 + 12x + 8x + 4x^2 = 48$

$4x^2 + 20x - 24 = 0$

$$x = \frac{-20 \pm \sqrt{400 + 384}}{8} = \frac{-20 \pm 28}{8} = \left\langle \begin{matrix} 1 \\ -6 \end{matrix} \right.$$ the width of the path is 1 metre

2. $x + \sqrt{3x + 10} = 6 \rightarrow \sqrt{3x + 10} = 6 - x$

$\left(\sqrt{3x+10}\right)^2 = (6 - x)^2 \Rightarrow 3x + 10 = 36 - 12x + x^2 \rightarrow x^2 - 15x + 26 = 0$

$$x = \frac{15 \pm \sqrt{225 - 104}}{2} = \frac{15 \pm 11}{2} = \left\langle \begin{matrix} 13 \\ 2 \end{matrix} \right.$$

Check: $x = 13 \rightarrow 13 + \sqrt{39 + 10} = 6 \rightarrow 13 + 7 = 6$ NO

$x = 2 \rightarrow 2 + \sqrt{6 + 10} = 6 \rightarrow 2 + 4 = 6\ \nu$

Solution: $x = 2$

3. $\dfrac{x + 5}{x + 3} = 1 - \dfrac{x^2 + 3x + 6}{x^2 + 2x - 3}$ factor: $x^2 + 2x - 3 = 0 \rightarrow x = \left\langle \begin{matrix} 1 \\ -3 \end{matrix} \right.$

$x^2 + 2x - 3 = (x - 1)(x + 3) \rightarrow LCF = (x - 1)(x + 3)$

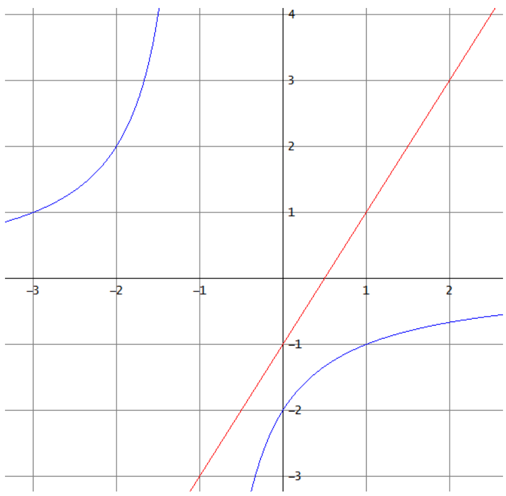
$$\frac{(x-1)(x+5)}{(x-1)(x+3)} = \frac{(x-1)(x+3)}{(x-1)(x+3)} - \frac{x^2 + 3x + 6}{(x-1)(x+3)} \rightarrow x^2 + 4x - 5 = x^2 + 2x - 3 - x^2 - 3x - 6$$

$$x^2 + 5x + 4 = 0 \rightarrow x = \frac{-5 \pm \sqrt{25 - 16}}{2} = \left\langle \begin{matrix} -1 \\ -4 \end{matrix} \right.$$

4. $$\left.\begin{matrix} 2x - y - 1 = 0 \\ y = -\dfrac{2}{x+1} \end{matrix}\right\} \rightarrow 2x + \frac{2}{x+1} - 1 = 0 \Rightarrow 2x(x+1) + 2 - (x+1) = 0$$

$2x^2 + x + 2 - x - 1 = 0 \Rightarrow 2x^2 + 1 = 0 \Rightarrow 2x^2 = -1$ No solution, inconsistent system

$$\left.\begin{matrix} y = 2x - 1 \\ y = -\dfrac{2}{x+1} \end{matrix}\right\} \left.\begin{matrix} \text{straight line} \\ \text{hyperbola} \end{matrix}\right\}$$



5. $\frac{(x-2)(x+2)}{4}-\frac{x-6}{2}\leq\frac{10+x}{5}\rightarrow\frac{x^2-4}{4}-\frac{x-6}{2}\leq\frac{10+x}{5}$

$\frac{5x^2-20}{20}-\frac{10x-60}{20}\leq\frac{40+4x}{20}\rightarrow 5x^2-20-10x+60\leq 40+4x$

$$5x^2-14x=0\rightarrow x(5x-14)=0\rightarrow\left\langle\begin{array}{l}x_1=0\\5x-14=0\rightarrow x_2=\frac{14}{5}\end{array}\right.$$

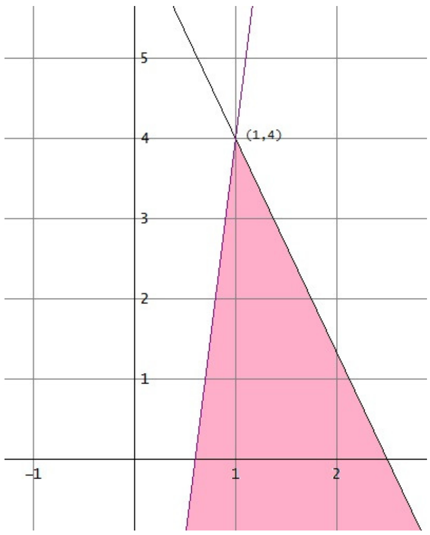
We study the sign:

Solution: $\left[0,\frac{14}{5}\right]$

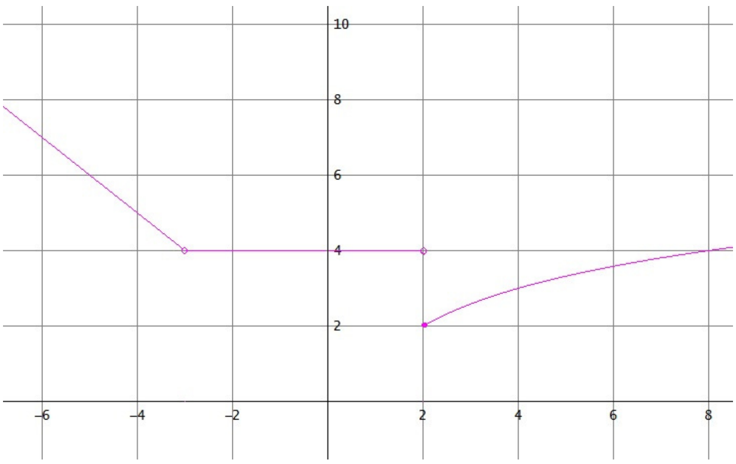
6. $\left.\begin{array}{l}2x+\frac{3y}{4}\leq 5\\10x-y>6\end{array}\right\}$ By graphing:

$y=\frac{20-8x}{3}$	x	1	-2
	y	4	12

$y=10x-6$	x	1	0
	y	4	-6



7. $f(x)=\begin{cases}1-x & x<-3\\4 & -3<x<2\\1+\log_2 x & x\geq 2\end{cases}$



Domain: $R-\{-3\}$

Range: $[2,+\infty)$

Removable discontinuity in x = -3 and jump discontinuity in x = 2

8.

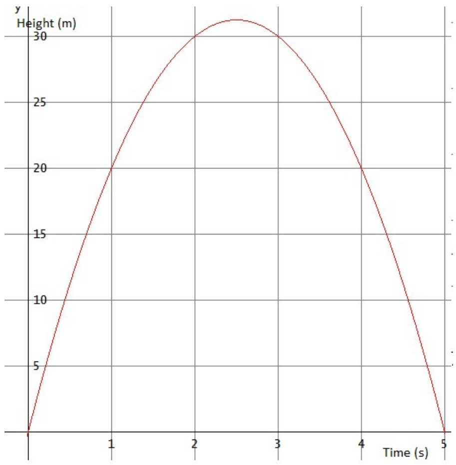
a) $3^{x-2} = \frac{1}{27} \Rightarrow 3^{x-2} = 3^{-3} \Rightarrow x-2=-3 \Rightarrow x=-1$

b) $log_{(2x+1)} 49 = 2 \Leftrightarrow (2x+1)^2 = 49 = 7^2 \Rightarrow 2x+1=7 \Rightarrow x=3$

9. A ball is thrown in the air so that t seconds after it is thrown, its height h metres above it starting point is given by the function $h(t) = 25t - 5t^2$.
Draw the graph of the function of $0 \le t \le 6$ and find:

a) The time when the ball is at its greatest height.
b) The greatest height reached by the ball.
c) The interval of time during which the ball is at a height of more than 30 m.

$y = -5x^2 + 25x$ it is a parabola $\cap$ with vertex $\rightarrow x = -\frac{25}{-10} = \frac{5}{2}$



So the ball is at its greatest height 2.5 seconds after it is thrown

The greatest height is for

$x = \frac{5}{2} \rightarrow y = -5\left(\frac{5}{2}\right)^2 + 25 \cdot \frac{5}{2} = \frac{125}{4} = 31.25\,m$

The ball is at a height of more than 30 metres between 2 and 3 seconds after the ball is thrown.

10. Graph A: It is a exponential function with base<1 (decreasing) and it pass through the point (-1, 2), so the function is $y = \left(\frac{1}{2}\right)^x$

Graph B: It is a radical function with Domain $[-1,+\infty)$ and negative. It pass through the point (3, -2), so the function is $y = -\sqrt{x+1}$

Graph C: It is a exponential function with, with base>1 (increasing). It pass through the point (1, 3), so the function is $y = 3^x$

Graph D: It is a logarithmic function with Domain $(-1,+\infty)$ (Type $log_a(x+1)$).
It pass through the points (0,0) and (2,1), so $y = log_2(x+1)$