EXAM 2_1 (Algebra: Inequalities, Word Problems)

1. Solve the following inequality and draw the solutions on the number line:
\[
\frac{3-x}{4} - \frac{x+1}{6} < 2 - \frac{2-x}{3}
\]
(1 point)

2. Solve the following systems of inequalities:
   (3 points)
   a) \[
   \begin{align*}
   \frac{x-15}{2} &\leq 5 - 2x \\
   2 - x &< \frac{1-x}{2}
   \end{align*}
   \]
   b) \[
   \begin{align*}
   2x - 3(x + 1) &\geq 2 \\
   -2(x - 2) + 5x &< 1
   \end{align*}
   \]
   c) \[
   \begin{align*}
   x - 2y &\geq 2 - y \\
   -3x &< 5 + y
   \end{align*}
   \]

3. Solve:
   (1.5 points)
   \[
   \begin{align*}
   (1 + x^2)y^2 &= 5 \\
   4x - y &= 0
   \end{align*}
   \]

4. Some kilograms of olive oil, which costs €5/kg, are combined with sunflower oil which costs €2.5/kg, to obtain 40 kg of a mixture which costs €3.25/kg How many kilograms of each type will we have to put into the mixture? (1.5 points)

5. A furniture shop sold a total of 315 sofas and tables. A sofa sold for €2300 and a table for €890 euro. The total sales were € 401 610. How many tables were sold? (1.5 points)

6. The area of a rectangle is 91 cm² and its perimeter is 40 cm. Find the base and the height of the rectangle. (1.5 points)
1. \[ \frac{3 - x}{4} - \frac{x + 1}{6} < 2 - \frac{2 - x}{3} \Rightarrow 9 - 3x - 2x - 2 < 24 - 8 + 4x \Rightarrow -9x < 9 \Rightarrow 9x > -9 \Rightarrow x > -1 \]

Solution: \((-1, +\infty)\)

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2. a) \[ \frac{x - 15}{2} \leq 5 - 2x \]

\[ 2 - x < \frac{1 - x}{2} \]

\[ x - 15 \leq 10 - 4x \]

\[ x + 4x \leq 10 + 15 \]

\[ x \leq 5 \]

\[ 4 - 2x < 1 - x \]

\[ -2x + x < 1 - 4 \]

\[ x > 3 \]

Solution: \((3, 5]\)

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b) \[ 2x - 3(x + 1) \geq 2 \]

\[ -2(x - 2) + 5x < 1 \]

\[ 2x - 3x - 3 \geq 2 \]

\[ -2x + 4 + 5x < 1 \]

\[ -x \geq 5 \]

\[ x \leq -5 \]

\[ 3x < -3 \]

\[ x < -1 \]

Solution: \(( -\infty, -5]\)

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c) \[ x - 2y \geq 2 - y \]

\[ -3x < 5 + y \]

\[ y = x - 2 \]

\[ -3x = 5 + y \]

\[ y = -3x - 5 \]

<table>
<thead>
<tr>
<th>[ y = x - 2 ]</th>
<th>x</th>
<th>0</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-2</td>
<td>0</td>
<td></td>
</tr>
</tbody>
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<table>
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<tr>
<th>[ y = -3x - 5 ]</th>
<th>x</th>
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</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-5</td>
<td>1</td>
<td></td>
</tr>
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Point (2,2)

\[ \frac{2 - 2 \cdot 2 \geq 2 - 2}{-3 \cdot 2 < 5 + 2} \]

\[ -2 \geq 0 \text{ NO} \]

\[ -6 < 7 \text{ SI} \]
3. \( (1 + x^2)y^2 = 5 \) \( 4x - y = 0 \) \( \Rightarrow \) \( y^2 + x^2y^2 = 5 \) \( y = 4x \) \( \Rightarrow (4x)^2 + x^2(4x)^2 = 5 \)

\( 16x^4 + 16x^2 - 5 = 0 \) \( \rightarrow z = x^2 \) \( \Rightarrow 16z^2 + 16z - 5 = 0 \)

\[
\begin{align*}
z &= \frac{-16 \pm \sqrt{256 + 320}}{32} = \frac{-16 \pm 24}{32} = \begin{cases} 
\frac{1}{4} \rightarrow x = \pm \sqrt{\frac{1}{4}} = \pm \frac{1}{2} \\
-\frac{5}{4} \rightarrow x = \pm \sqrt{-\frac{5}{4}} \rightarrow \text{NO}
\end{cases} \\
x &= \frac{1}{2} \rightarrow y = 4 \cdot \frac{1}{2} = 2, \ x = -\frac{1}{2} \rightarrow y = 4 \cdot -\frac{1}{2} = -2
\end{align*}
\]
Solution: \[
\begin{align*}
\frac{x}{2} \quad & y = 2 \\
\frac{1}{2} \quad & y = -2
\end{align*}
\]

4. Some kilograms of olive oil, which costs €5/kg, are combined with sunflower oil which costs €2.5/kg, to obtain 40 kg of a mixture which costs €3.25/kg How many kilograms of each type will we have to put into the mixture?

<table>
<thead>
<tr>
<th></th>
<th>Olive oil</th>
<th>Sunflower oil</th>
<th>Mixture</th>
</tr>
</thead>
<tbody>
<tr>
<td>kilograms</td>
<td>x</td>
<td>40 - x</td>
<td>40</td>
</tr>
<tr>
<td>Price/kg</td>
<td>5</td>
<td>2.5</td>
<td>3.25</td>
</tr>
</tbody>
</table>

Equation: 
\[5x + 2.5(40 - x) = 3.25 \cdot 40 \Rightarrow 5x + 100 - 2.5x = 130 \Rightarrow 2.5x = 30 \Rightarrow x = 12\]

Answer- We needed 12 kg of olive oil and 28 kg of sunflower oil

5. A furniture shop sold a total of 315 sofas and tables. A sofa sold for €2300 and a table for €890 euro. The total sales were € 401 610. How many tables were sold?

Number of sofas \(x\); Number of tables \(y\)

\[
\begin{align*}
x + y &= 315 \\
2300x + 890y &= 401610
\end{align*}
\]

\[
\begin{align*}
230x + 89(315 - x) &= 401610 \\
230x - 89x &= 401610 - 28035
\end{align*}
\]

\[141x = 12126 \Rightarrow x = 86 \Rightarrow y = 315 - x = 315 - 86 = 229\]

Answer: They have sold 86 sofas and 229 tables

6. The area of a rectangle is 91 cm\(^2\) and its perimeter is 40 cm. Find the base and the height of the rectangle.

Base \(x\) Height \(y\)

Equations: 
\[
\begin{align*}
x \cdot y &= 91 \\
2x + 2y &= 40
\end{align*}
\]

\[
\begin{align*}
x \cdot y &= 91 \\
y &= \frac{40 - 2x}{2} = 20 - x
\end{align*}
\]

\[x^2 - 20x + 91 = 0 \Rightarrow x = \frac{20 \pm \sqrt{400 - 364}}{2} = \frac{13 \Rightarrow y = 20 - 13 = 7}{7 \Rightarrow y = 20 - 7 = 13}\]

Answer: Base 13 cm and height 7 cm