Maths 4th ESO

EXAM 1_1 (Real Numbers- Surds - Polynomials)

1) Classify according to number type and mark on the real number line the following. (Notice that some numbers may be of more than one type).

(2 points)

a) $-\sqrt{16}$

b) 1.75

c) $\sqrt{5}$

d) $-0.\widehat{6}$

2) Work out and simplify: (1.5 points)

a) $x(x+1)^2 - x^2(x+1) - \left(x+\frac{1}{2}\right)\left(x-\frac{1}{2}\right) =$

b) $(x^2-3)(x^3-2x^2+x-2) =$

c) $x^3y^2(2x^2y^2-3xy) - 3x^2y^3(1-x)(1+x) =$

3) Rationalise and simplify: (2 points)

a) $\dfrac{\sqrt{3}+1}{2\sqrt{3}}$

b) $\dfrac{10}{\sqrt[5]{5^3}}$

c) $\dfrac{\sqrt{3}+\sqrt{5}}{\sqrt{3}-\sqrt{5}}$

5) Work out and simplify: (3 points)

a) $\sqrt{2\sqrt[3]{\sqrt{8}}} =$

b) $\sqrt{2}.\sqrt[3]{16}\sqrt[3]{32} =$

c) $3\sqrt[3]{81} - 2\sqrt[6]{3^2} + 3\sqrt[3]{\dfrac{3}{8}} =$

d) $\dfrac{\sqrt{24}-\sqrt{150}+4\sqrt{54}}{\sqrt{6}} =$

6) Calculate quotient and remainder in the following divisions: (1.5 points)

a) $(6x^3-2x^2-1) \div (x^2+x+2)$

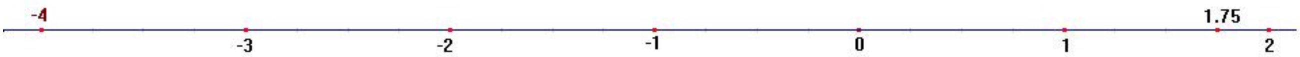
b) $(x^5-2x^2+3) \div (x+1)$

SOLUTION

1) Classify according to number type and mark on the real number line the following. (Notice that some numbers may be of more than one type).

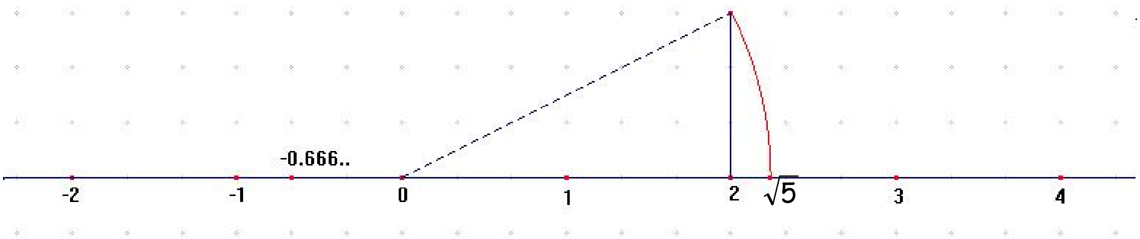
a) $-\sqrt{16} = -4$ Integer number, Rational

b) $1.75 = \frac{7}{4}$ Terminating decimal, Rational $1.75 = \frac{175}{100} = \frac{7}{4}$



c) $\sqrt{5}$ Irrational

d) $-0.\hat{6} = -\frac{2}{3}$ Repeating decimal, Rational $\left.\begin{matrix} N = 0.6666... \\ 10N = 6.666... \end{matrix}\right\} 9N = 6 \rightarrow N = \frac{2}{3}$



2) Work out and simplify:

a) $x(x+1)^2 - x^2(x+1) - \left(x+\frac{1}{2}\right)\left(x-\frac{1}{2}\right) = x(x^2+2x+1) - x^3 - x^2 - \left(x^2 - \frac{1}{4}\right) =$

$= x^3 + 2x^2 + x - x^3 - x^2 - x^2 + \frac{1}{4} = x + \frac{1}{4}$

b) $(x^2-3)(x^3-2x^2+x-2) = x^5 - 2x^4 + x^3 - 2x^2 - 3x^3 + 6x^2 - 3x + 6 =$

$= x^5 - 2x^4 - 2x^3 + 4x^2 - 3x + 6$

c) $x^3y^2(2x^2y^2 - 3xy) - 3x^2y^3(1-x)(1+x) = 2x^5y^4 - 3x^4y^3 - 3x^2y^3(1-x^2) =$

$= 2x^5y^4 - 3x^4y^3 - 3x^2y^3 + 3x^4y^3 = 2x^5y^4 - 3x^2y^3$

3) Rationalise and simplify:

a) $\frac{\sqrt{3}+1}{2\sqrt{3}} = \frac{(\sqrt{3}+1)\sqrt{3}}{2\sqrt{3}\sqrt{3}} = \frac{3+\sqrt{3}}{6}$

b) $\frac{10}{\sqrt[5]{5^3}} = \frac{10\sqrt[5]{5^2}}{\sqrt[5]{5^3}\sqrt[5]{5^2}} = \frac{10\sqrt[5]{5^2}}{5} = 2\sqrt[5]{25}$

c) $\frac{\sqrt{3}+\sqrt{5}}{\sqrt{3}-\sqrt{5}} = \frac{(\sqrt{3}+\sqrt{5})(\sqrt{3}+\sqrt{5})}{(\sqrt{3}-\sqrt{5})(\sqrt{3}+\sqrt{5})} = \frac{(\sqrt{3}+\sqrt{5})^2}{\sqrt{3}^2-\sqrt{5}^2} = \frac{3+5+2\sqrt{3}\sqrt{5}}{3-5} =$

$= \frac{8+2\sqrt{15}}{-2} = \frac{2(4+2\sqrt{15})}{-2} = -4-\sqrt{15}$

4) Work out and simplify:

a) $\sqrt{2\sqrt[3]{\sqrt{8}}} = \sqrt{2\sqrt[6]{2^3}} = \sqrt{\sqrt[6]{2^6\cdot 2^3}} = \sqrt[12]{2^9} = \sqrt[4]{2^3}$

b) $\sqrt{2}.\sqrt[3]{16}\sqrt[3]{32} = \sqrt[6]{2^3}.\sqrt[6]{16^2}\cdot\sqrt[6]{32^2} = \sqrt[6]{2^3\cdot 2^8\cdot 2^{10}} = \sqrt[6]{2^{21}} = 2^3\sqrt[6]{2^3} = 2^3\sqrt{2} = 8\sqrt{2}$

c) $3\sqrt[3]{81} - 2\sqrt[6]{3^2} + 3\sqrt[3]{\frac{3}{8}} = 3\sqrt[3]{3^4} - 2\sqrt[3]{3} + 3\sqrt[3]{\frac{3}{2^3}} = 9\sqrt[3]{3} - 2\sqrt[3]{3} + \frac{3}{2}\sqrt[3]{3} = \frac{17}{2}\sqrt[3]{3}$

d) $\frac{\sqrt{24}-\sqrt{150}+4\sqrt{54}}{\sqrt{6}} = \frac{\sqrt{2^3\cdot 3}-\sqrt{2\cdot 3\cdot 5^2}+4\sqrt{2\cdot 3^3}}{\sqrt{6}} =$

$\frac{\sqrt{24}-\sqrt{150}+4\sqrt{54}}{\sqrt{6}} = \frac{9\sqrt{6}}{\sqrt{6}} = 9$

5) Calculate quotient and remainder in the following divisions:

a) $(6x^3-2x^2-1)\div(x^2+x+2)$

$6x^3$	$-2x^2$		-1	x^2	$+x$	$+2$
$-6x^3$	$-6x^2$	$-12x$		$6x$	-8	
	$-8x^2$	$-12x$	-1			
	$+8x^2$	$+8x$	$+16$			
		$-4x$	$+15$			

Quotient: $6x-8$ Remainder: $-4x+15$

b) $(x^5-2x^2+3)\div(x+1)$ Ruffini's rule

	1	0	0	-2	0	+3
-1		-1	+1	-1	+3	-3
	1	-1	+1	-1	+3	0

Quotient: $x^4-x^3+x^2-x+3$ Remainder: 0