



## EXAM 1\_1 (Real numbers)

Name: \_\_\_\_\_ Group: \_\_\_\_\_

1. Use your calculator to work out the following calculations. Express the results in scientific notation. (When you obtain results of more than 3 decimal figures, round them to three decimal places) (1 point)

a)  $(1.22 \cdot 10^{10}) \div (3.305 \cdot 10^{-4}) =$

b)  $\sqrt[5]{7.6318 \cdot 10^{-12}} =$

2. Calculate and simplify (write the steps you have taken to reach the solution):

(4.5 points)

a)  $\frac{\sqrt{2} \cdot \sqrt[4]{20}}{\sqrt[4]{8}} =$

b)  $(3\sqrt{20} + 2\sqrt{80} - 4\sqrt{125}) \div 3\sqrt{500} =$

c)  $2\sqrt[3]{81} - \sqrt[3]{24} + 3\sqrt[3]{108} - \sqrt[3]{256} =$

d)  $\sqrt{\sqrt[4]{a^2} \cdot \sqrt[3]{a^2}} =$

e)  $(a - 2\sqrt{b})^2 - (a + 2\sqrt{b})^2 =$

f)  $4\sqrt{\frac{75}{4}} + 2\sqrt{3} - \frac{7}{3}\sqrt{27} + \sqrt{\frac{48}{9}} =$

3. Classify according to number type and mark on the real number line the following. (Notice that some numbers may be of more than one type).

(1.5 points)

a)  $-\sqrt{11}$

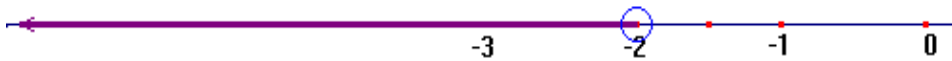
b)  $\sqrt{9}$

c)  $-0.8\bar{3}$

4. Use both interval and set notation to describe the interval shown on the graph:

(1.5 points)

a)



b)



c)



5. Rationalize and simplify (write the steps you have taken to reach the solution):

(1.5 points)

$$\frac{\sqrt{2}}{\sqrt{5} + \sqrt{2}} - \frac{\sqrt{2}}{\sqrt{5} - \sqrt{2}}$$

**SOLUTION**

1. Use your calculator to work out the following calculations. Express the results in scientific notation. (When you obtain results of more than 3 decimal figures, round them to three decimal places)

$$c) (1.22 \cdot 10^{10}) \div (3.305 \cdot 10^{-4}) = 3.691 \cdot 10^{13}$$

$$d) \sqrt[5]{7.6318 \cdot 10^{-12}} = 0.0059777579 \approx 5.978 \cdot 10^{-3}$$

2. Calculate and simplify (write the steps you have taken to reach the solution):

$$a) \frac{\sqrt{2} \cdot \sqrt[4]{20}}{\sqrt[4]{8}} = \frac{\sqrt[4]{2^2} \cdot \sqrt[4]{2^2 \cdot 5}}{\sqrt[4]{2^3}} = \sqrt[4]{\frac{2^4 \cdot 5}{2^3}} = \sqrt[4]{10}$$

$$b) (3\sqrt{20} + 2\sqrt{80} - 4\sqrt{125}) \div 3\sqrt{500} = \frac{3\sqrt{2^2 \cdot 5} + 2\sqrt{2^4 \cdot 5} - 4\sqrt{5^3}}{3\sqrt{2^2 \cdot 5^3}} =$$

$$= \frac{6\sqrt{5} + 8\sqrt{5} - 20\sqrt{5}}{30\sqrt{5}} = \frac{-6\sqrt{5}}{30\sqrt{5}} = -\frac{6}{30} = -\frac{1}{5}$$

$$c) 2\sqrt[3]{81} - \sqrt[3]{24} + 3\sqrt[3]{108} - \sqrt[3]{256} = 2\sqrt[3]{3^4} - \sqrt[3]{2^3 \cdot 3} + 3\sqrt[3]{2^2 \cdot 3^3} - \sqrt[3]{2^8} =$$

$$6\sqrt[3]{3} - 2\sqrt[3]{3} + 9\sqrt[3]{4} - 4\sqrt[3]{4} = 4\sqrt[3]{3} + 5\sqrt[3]{4}$$

$$d) \sqrt[4]{a^2 \cdot \sqrt[3]{a^2}} = \sqrt[4]{a^2 \cdot \sqrt[3]{a^2}} = \sqrt[8]{3a^6 \cdot a^2} = \sqrt[8]{a^8} = \sqrt[3]{a}$$

$$e) (a - 2\sqrt{b})^2 - (a + 2\sqrt{b})^2 = a^2 - 4a\sqrt{b} + (2\sqrt{b})^2 - (a^2 + 4a\sqrt{b} + (2\sqrt{b})^2) =$$

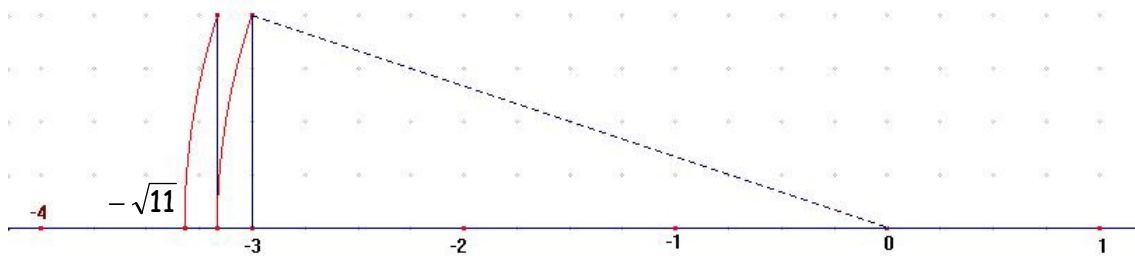
$$= a^2 - 4a\sqrt{b} + 4b - a^2 - 4a\sqrt{b} - 4b = -8a\sqrt{b}$$

$$f) 4\sqrt{\frac{75}{4}} + 2\sqrt{3} - \frac{7}{3}\sqrt{27} + \sqrt{\frac{48}{9}} = 4\sqrt{\frac{3 \cdot 5^2}{2^2}} + 2\sqrt{3} - \frac{7}{3}\sqrt{3^3} + \sqrt{\frac{2^4 \cdot 3}{3^2}} =$$

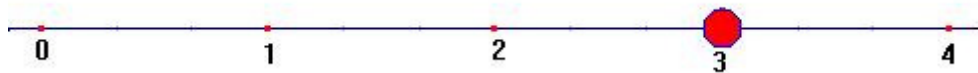
$$= \frac{20}{2}\sqrt{3} + 2\sqrt{3} - \frac{21}{3}\sqrt{3} + \frac{4}{3}\sqrt{3} = \left(10 + 2 - \frac{21}{3} + \frac{4}{3}\right)\sqrt{3} = \frac{19}{3}\sqrt{3}$$

3. Classify according to number type and mark on the real number line the following. (Notice that some numbers may be of more than one type).

a)  $-\sqrt{11}$ , real, irrational



b)  $\sqrt{9} = 3$  real, whole number, natural

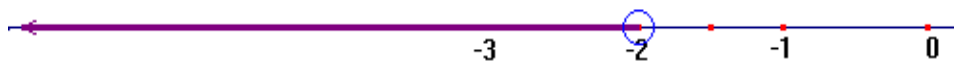


c)  $-0.8\bar{3} = -\frac{5}{6}$  real, rational, recurring decimal



4. Use both interval and set notation to describe the interval shown on the graph:

a) Interval  $(-\infty, -2)$ ; set  $\{x \in \mathbb{R} / x < -2\}$



b) Interval  $[-1, 4)$ ; set  $\{x \in \mathbb{R} / -1 \leq x < 4\}$



c) Interval  $[-2, +\infty)$ ; set  $\{x \in \mathbb{R} / x \geq -2\}$



5. Rationalize and simplify:

$$\frac{\sqrt{2}}{\sqrt{5} + \sqrt{2}} - \frac{\sqrt{2}}{\sqrt{5} - \sqrt{2}} = \frac{\sqrt{10} - 2}{3} - \frac{\sqrt{10} + 2}{3} = -\frac{4}{3}$$

$$\frac{\sqrt{2}}{\sqrt{5} + \sqrt{2}} = \frac{\sqrt{2}(\sqrt{5} - \sqrt{2})}{(\sqrt{5} + \sqrt{2})(\sqrt{5} - \sqrt{2})} = \frac{\sqrt{2}(\sqrt{5} - \sqrt{2})}{(\sqrt{5})^2 - (\sqrt{2})^2} = \frac{\sqrt{10} - 2}{3}$$

$$\frac{\sqrt{2}}{\sqrt{5} - \sqrt{2}} = \frac{\sqrt{2}(\sqrt{5} + \sqrt{2})}{(\sqrt{5} + \sqrt{2})(\sqrt{5} - \sqrt{2})} = \frac{\sqrt{2}(\sqrt{5} + \sqrt{2})}{(\sqrt{5})^2 - (\sqrt{2})^2} = \frac{\sqrt{10} + 2}{3}$$