EXAM 1_2 (Polynomials- Surds)

1. Work out and simplify:
   a) (1 p) \( x(x + 2)^2 - x^2(x + 4) - 2(x + 2)(x - 2) = \)
   b) (0.75 p) \( (x^2 - 2)(x^3 - x^2 + 2x - 1) = \)
   c) (0.75 p) \( 2x^3y^2(x^2y^2 - 3xy) - 2x^2(x^2y^3 + 2) = \)

2. Rationalise and simplify:
   a) (0.75 p) \( \frac{\sqrt{2}}{3 - \sqrt{2}} = \)
   b) (0.75 p) \( \frac{2a}{\sqrt{3}a^2} = \)
   c) (0.75 p) \( \frac{\sqrt{5} + \sqrt{2}}{\sqrt{5} - \sqrt{2}} = \)

3. Work out and simplify:
   a) (0.75 p) \( \sqrt{2500} = \)
   b) (0.75 p) \( \sqrt{2 \cdot 8 \cdot 3 \cdot 16} = \)
   c) (0.75 p) \( 7 \cdot \frac{2}{\sqrt{3}} - 3 \cdot \frac{\sqrt{50}}{\sqrt{12}} + \frac{2}{\sqrt{27}} - \frac{1}{5} \cdot \frac{8}{\sqrt{3}} = \)

4. Factorise:
   a) (0.5 p) \( 16x^4 - 8x^3 + x^2 = \)
   b) (1.25 p) \( 5x^3 - 20x^2 - 20x + 80 = \)
   cc) (1.25 p) \( x^4 - 5x^2 + 4 = \)
1. Work out and simplify:

a) \(x(x + 2)^2 - x^2(x + 4) - 2(x + 2)(x - 2) = x(x^2 + 4x + 4) - x^3 - 4x^2 - 2(x^2 - 4) = x^3 + 4x^2 + 4x - x^3 - 4x^2 - 2x^2 + 8 = -2x^2 + 4x + 8\)

b) \((x^2 - 2)(x^3 - x^2 + 2x - 1) = x^5 - x^4 + 2x^3 - x^2 - 2x^3 + 2x^2 - 4x + 2 = x^5 - x^4 + x^2 - 4x + 2\)

c) \(2x^3y^2(x^2y^2 - 3xy) - 2x^2(x^2y^3 + 2) = 2x^5y^4 - 6x^4y^3 - 2x^4y^3 - 4x^2 = 2x^5y^4 - 8x^4y^3 - 4x^2\)

2. Rationalise and simplify:

a) \(\frac{\sqrt{2}}{3 - \sqrt{2}} = \frac{\sqrt{2}(3 + \sqrt{2})}{(3 - \sqrt{2})(3 + \sqrt{2})} = \frac{3\sqrt{2} + 2}{7}\)

b) \(\frac{2a}{\sqrt[3]{a^2}} = \frac{2a\sqrt[3]{a}}{\sqrt[3]{a^2}\sqrt[3]{a}} = \frac{2a\sqrt[3]{a}}{a} = 2\sqrt[3]{a}\)

c) \(\frac{\sqrt{5} + \sqrt{2}}{\sqrt{5} - \sqrt{2}} = \frac{(\sqrt{5} + \sqrt{2})(\sqrt{5} + \sqrt{2})}{(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})} = \frac{5 + 2\sqrt{5\sqrt{2}} + 2}{5 - 2} = \frac{7 + 2\sqrt{10}}{3}\)

3. Work out and simplify:

a) \(\sqrt{2500} = \sqrt[4]{2^2 \cdot 5^4} = 5\sqrt{2^2} = 5\sqrt{2}\)

b) \(\sqrt{2} \cdot \sqrt[3]{32} \cdot \sqrt{16} = \sqrt[6]{2^3 \cdot 2^3} = \sqrt[6]{2^6} \cdot 2^9 \cdot 2^8 = 2^{20} = 4\sqrt{2}\)

c) \(7\sqrt[3]{\frac{2}{3}} - 3\sqrt[3]{\frac{50}{12}} + \sqrt[3]{\frac{2}{27}} - \frac{1}{5}\sqrt[3]{8} = 7\sqrt[3]{\frac{2}{3}} - 3\cdot\frac{2}{3}\sqrt[3]{\frac{50}{12}} + \frac{1}{3}\sqrt[3]{2} - \frac{2}{5}\sqrt[3]{3} = (7 - \frac{15}{2} + \frac{1}{3} - \frac{2}{5})\sqrt[3]{\frac{2}{3}} = \left(\frac{210 - 225 + 10 - 12}{30}\right)\sqrt[3]{\frac{2}{3}} = \frac{17}{30}\sqrt[3]{\frac{2}{3}}\)
4. Factorise:

a) \(16x^4 - 8x^3 + x^2 = x^2(16x^2 - 8x + 1) = x^2(4x - 1)^2\)

b) \(5x^3 - 20x^2 - 20x + 80 = 5(x^3 - 4x^2 - 4x + 16)\)

\(Div(16) = \pm 1, \pm 2, \pm 4, \pm 8, \pm 16\)

\(P(1) = 1 - 4 - 4 + 16 \neq 0 \quad P(-1) = -1 - 4 + 4 + 16 \neq 0\)

\(P(2) = 8 - 16 - 8 + 16 = 0\)

\[
\begin{array}{c|cccc}
2 & 1 & -4 & -4 & +16 \\
\hline
1 & 2 & -4 & -16 & \\
\end{array}
\]

\(5x^3 - 20x^2 - 20x + 80 = 5(x - 2)(x + 2)(x - 4)\)

c) \(x^4 - 5x^2 + 4 \quad Div(4) = \pm 1, \pm 2, \pm 4\)

\(P(1) = 1 - 5 + 4 = 0 \quad P(-1) = 1 - 5 + 4 = 0\)

\[
\begin{array}{c|cccc}
1 & 1 & 0 & -5 & 0 & +4 \\
\hline
1 & 1 & 1 & -4 & -4 & \\
\end{array}
\]

\(x^2 - 4 = (x - 2)(x + 2)\)

\[
\begin{array}{c|cccc}
-1 & 1 & 0 & +4 & \\
\hline
1 & 0 & -4 & 0 & \\
\end{array}
\]

\(x^4 - 5x^2 + 4 = (x - 1)(x + 1)(x - 2)(x + 2)\)