Maths 4^{th} ESO

EXAM 3_3 (Trigonometry- Equations)

Name:………………………………………………………………………………………………………

1. Suppose that $tan\,\alpha = \frac{3}{4}$ and α lies in quadrant III. Find the other trigonometric ratios for α. Draw the angle α. (2 points)

2. In a right-angled triangle the length of a leg is twice the other. Calculate the trigonometric ratios of the smallest angle. (1.75 points)

3. A woodcutter wants to determine the height of a tall tree. He stands at some distance from the tree and determines that the angle of elevation to the top of the tree is 40°. He moves 30 metres closer to the tree, and now the angle of elevation is 50°. If the woodcutter's eyes are 1.5m above the ground, how tall is the tree? (2 points)

4. In an isosceles triangle, the base is 12 metres long and the congruent angles are 70° each. Find the length of the other sides and the area. (2 points)

5. Solve: (2.25 points)

a) $x + \sqrt{3x+10} = 6$

b) $x + \frac{1}{x} = \frac{4}{\sqrt{3}}$

c) $(2x^4 - 3x^2 - 20)(3x-2) = 0$

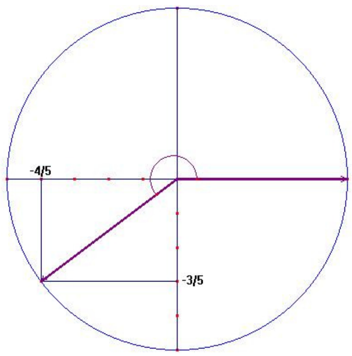
SOLUTION

1. Suppose that $tan\,\alpha = \frac{3}{4}$ and α lies in quadrant III. Find the other trigonometric ratios for α. Draw the angle α.

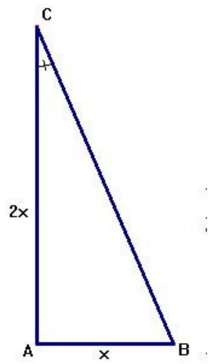
$$1 + tan^2\,\alpha = \frac{1}{cos^2\,\alpha} \rightarrow 1 + \left(\frac{3}{4}\right)^2 = \frac{1}{cos^2\,\alpha} \rightarrow \frac{25}{16} = \frac{1}{cos^2\,\alpha} \rightarrow cos^2\,\alpha = \frac{16}{25}$$

$$cos\,\alpha = \pm\sqrt{\frac{16}{25}} \rightarrow cos\,\alpha = -\frac{4}{5} \rightarrow tan\,\alpha = \frac{sin\,\alpha}{cos\,\alpha} \rightarrow sin\,\alpha = tan\,\alpha \cdot cos\,\alpha$$

$$sin\,\alpha = \frac{3}{4} \cdot -\frac{4}{5} = -\frac{3}{5}$$



2. In a right-angled triangle the length of a leg is twice the other. Calculate the trigonometric ratios of the smallest angle. $h^2 = x^2 + (2x)^2 = 5x^2 \rightarrow h = \sqrt{5}x$

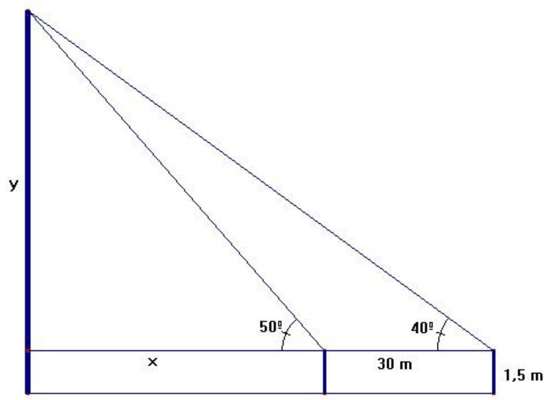


$$sin\,\alpha = \frac{x}{\sqrt{5}x} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

$$cos\,\alpha = \frac{2x}{\sqrt{5}x} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$tan\,\alpha = \frac{x}{2x} = \frac{1}{2}$$

3. A woodcutter wants to determine the height of a tall tree. He stands at some distance from the tree and determines that the angle of elevation to the top of the tree is 40°. He moves 30 metres closer to the tree, and now the angle of elevation is 50°. If the woodcutter's eyes are 1.5m above the ground, how tall is the tree?



$$\left.\begin{array}{l} tan\,40 = \dfrac{y}{x+30} \\ tan\,50 = \dfrac{y}{x} \end{array}\right\} \left.\begin{array}{l} y = (x+30)\,tan\,40 \\ y = x\,tan\,50 \end{array}\right\}$$

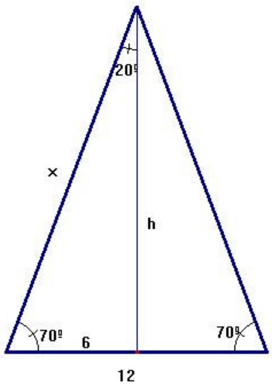
$$x\,tan\,40 + 30\,tan\,40 = x\,tan\,50$$

$$x(tan\,50 - tan\,40) = 30\,tan\,40$$

$$x = 71.38m \rightarrow y = 71.38 \cdot tan\,50 = 85.07m$$

The tree is 85.07+1.5=86.57 metres tall

4. In an isosceles triangle, the base is 12 metres long and the congruent angles are 70° each. Find the length of the other sides and the area.



$$cos\,70 = \frac{6}{x} \Rightarrow x = \frac{6}{cos\,70} = 17.54\,m$$

The other sides are 17.54 metres long

$$tan\,70 = \frac{h}{6} \Rightarrow h = 6\,tan\,70 = 16.48\,m$$

Area: $A = \dfrac{12 \cdot 16.48}{2} = 98.9\,m^2$

5. Solve:

a) $x + \sqrt{3x+10} = 6 \to \sqrt{3x+10} = 6 - x \Rightarrow 3x + 10 = 36 - 12x + x^2$

$$x^2 - 15x + 26 = 0 \to x = \frac{15 \pm \sqrt{121}}{2} = \left\langle \begin{array}{l} 13 \\ 2 \end{array} \right.$$

Checking: $\begin{array}{l} x = 13 \to \sqrt{39+10} = 6 - 13 \Rightarrow 7 = -7 \text{ NO} \\ x = 2 \to \sqrt{6+10} = 6 - 4 \Rightarrow 4 = 4 \text{ YES} \end{array}$ **Sol: x = 2**

b) $x + \dfrac{1}{x} = \dfrac{4}{\sqrt{3}} \to \sqrt{3}x^2 + \sqrt{3} = 4x \to \sqrt{3}x^2 - 4x + \sqrt{3} = 0$

$$x = \frac{4 \pm \sqrt{16 - 4\sqrt{3}\sqrt{3}}}{2\sqrt{3}} = \frac{4 \pm 2}{2\sqrt{3}} \left\langle \begin{array}{l} \dfrac{6}{2\sqrt{3}} = \dfrac{3}{\sqrt{3}} = \sqrt{3} \\ \dfrac{2}{2\sqrt{3}} = \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{3} \end{array} \right.$$

c) $(2x^4 - 3x^2 - 20)(3x - 12) = 0 \to \left\langle \begin{array}{l} 2x^4 - 3x^2 - 20 = 0 \to (*) \\ 3x - 12 = 0 \to x = 4 \end{array} \right.$

$$(*) 2x^4 - 3x^2 - 20 = 0, z = x^2 \to 2z^2 - 3z - 20 = 0 \to z = \frac{3 \pm \sqrt{169}}{4} = \left\langle \begin{array}{l} 4 \\ -\dfrac{5}{2} \end{array} \right.$$

$$x^2 = z = \left\langle \begin{array}{l} 4 \to x = \pm\sqrt{4} = \pm 2 \\ -\dfrac{5}{2} \to x = \pm\sqrt{-\dfrac{5}{2}} \text{ NO} \end{array} \right.$$

Solution: x = 4 , x = 2, x = -2