

SURDS 2

1. Work out and simplify:

a) $\sqrt{12} - 2\sqrt{75} + 3\sqrt{5} - 4\sqrt{20} =$

b) $\sqrt{2 + \sqrt{1 + \sqrt{9}}} =$

c) $\sqrt{2} \cdot \sqrt{8} \cdot \sqrt[3]{16} =$

d) $\sqrt{\sqrt{2500}} =$

e) $\sqrt{50} \cdot \sqrt{26} \cdot \sqrt{13} =$

f) $\sqrt{18a} - \sqrt{128a} + \sqrt{450a} - \sqrt{72a} =$

g) $7\sqrt{\frac{2}{3}} - 3\sqrt{\frac{25}{4}} + \sqrt{\frac{2}{27}} - \frac{1}{5}\sqrt{\frac{8}{3}} =$

h) $(3\sqrt{2} - 4\sqrt{8} + 5\sqrt{50}) \div (2\sqrt{2}) =$

i) $\frac{\sqrt{8} \cdot \sqrt[3]{6} \cdot \sqrt[6]{9}}{\sqrt{32}} =$

2. Rationalise:

a) $\frac{3}{\sqrt{5}} =$

b) $\frac{2}{\sqrt{5} - \sqrt{3}} =$

c) $\frac{1}{1 + \sqrt{2}} =$

d) $\frac{3 + \sqrt{2}}{\sqrt{2}} =$

e) $\frac{\sqrt{2}}{3 - \sqrt{2}} =$

SOLUTION

1. Work out and simplify:

$$\begin{aligned} \text{a) } \sqrt{12} - 2\sqrt{75} + 3\sqrt{5} - 4\sqrt{20} &= \sqrt{2^2 \cdot 3} - 2\sqrt{3 \cdot 5^2} + 3\sqrt{5} - 4\sqrt{2^2 \cdot 5} = \\ &= 2\sqrt{3} - 2 \cdot 5\sqrt{3} + 3\sqrt{5} - 4 \cdot 2\sqrt{5} = -8\sqrt{3} - 5\sqrt{5} \end{aligned}$$

$$\text{b) } \sqrt{2 + \sqrt{1 + \sqrt{9}}} = \sqrt{2 + \sqrt{1 + 3}} = \sqrt{2 + \sqrt{4}} = \sqrt{2 + 2} = \sqrt{4} = 2$$

$$\text{c) } \sqrt{2} \cdot \sqrt{8} \cdot \sqrt[3]{16} = \sqrt[6]{2^3} \cdot \sqrt[6]{8^3} \cdot \sqrt[6]{16^2} = \sqrt[6]{2^3 \cdot (2^3)^3 \cdot (2^4)^2} = \sqrt[6]{2^{20}} = \sqrt[3]{2^{10}} = 2^3 \sqrt[3]{2}$$

$$\text{d) } \sqrt{\sqrt{2500}} = \sqrt[4]{5^4 \cdot 2^2} = 5\sqrt[4]{2^2} = 5\sqrt{2}$$

$$\text{e) } \sqrt{50} \cdot \sqrt{26} \cdot \sqrt{13} = \sqrt{2 \cdot 5^2 \cdot 2 \cdot 13 \cdot 13} = \sqrt{2^2 \cdot 5^2 \cdot 13^2} = 2 \cdot 5 \cdot 13 = 130$$

$$\begin{aligned} \text{f) } \sqrt{18a} - \sqrt{128a} + \sqrt{450a} - \sqrt{72a} &= \sqrt{2 \cdot 3^2 a} - \sqrt{2^7 a} + \sqrt{2 \cdot 3^2 5^2 a} - \sqrt{2^3 3^2 a} = \\ &= 3\sqrt{2a} - 2^3 \sqrt{2a} + 3 \cdot 5\sqrt{2a} - 2 \cdot 3\sqrt{2a} = (3 - 8 + 15 - 6)\sqrt{2a} = 4\sqrt{2a} \end{aligned}$$

$$\begin{aligned} \text{g) } 7\sqrt{\frac{2}{3}} - 3\sqrt{\frac{25}{4}} + \sqrt{\frac{2}{27}} - \frac{1}{5}\sqrt{\frac{8}{3}} &= 7\sqrt{\frac{2}{3}} - 3\sqrt{\frac{5^2}{2^2}} + \sqrt{\frac{2}{3^3}} - \frac{1}{5}\sqrt{\frac{2^3}{3}} = \\ &= 7\sqrt{\frac{2}{3}} - 3 \cdot \frac{5}{2} + \frac{1}{3}\sqrt{\frac{2}{3}} - \frac{2}{5}\sqrt{\frac{2}{3}} = \left(7 + \frac{1}{3} - \frac{2}{5}\right)\sqrt{\frac{2}{3}} - \frac{15}{2} = \frac{104}{15}\sqrt{\frac{2}{3}} - \frac{15}{2} \end{aligned}$$

$$\begin{aligned} \text{h) } (3\sqrt{2} - 4\sqrt{8} + 5\sqrt{50}) \div (2\sqrt{2}) &= (3\sqrt{2} - 4 \cdot 2\sqrt{2} - 5 \cdot 5\sqrt{2}) \div 2\sqrt{2} = \\ &= (3\sqrt{2} - 8\sqrt{2} - 25\sqrt{2}) \div 2\sqrt{2} = (-30\sqrt{2}) \div 2\sqrt{2} = -15 \end{aligned}$$

$$\text{i) } \frac{\sqrt{8} \cdot \sqrt[3]{6} \cdot \sqrt[6]{9}}{\sqrt{32}} = \frac{\sqrt[6]{(2^3)^3} \cdot \sqrt[6]{(2 \cdot 3)^2} \cdot \sqrt[6]{3^2}}{\sqrt[6]{(2^5)^3}} = \sqrt[6]{\frac{2^9 2^2 3^4}{2^{15}}} = \sqrt[6]{\frac{3^4}{2^4}} = \sqrt[3]{\frac{9}{4}}$$

2. Rationalise:

$$\text{a) } \frac{3}{\sqrt{5}} = \frac{3\sqrt{5}}{(\sqrt{5})^2} = \frac{3\sqrt{5}}{5}$$

$$\text{b) } \frac{2}{\sqrt{5} - \sqrt{3}} = \frac{2(\sqrt{5} + \sqrt{3})}{(\sqrt{5} - \sqrt{3})(\sqrt{5} + \sqrt{3})} = \frac{2(\sqrt{5} + \sqrt{3})}{5 - 3} = \sqrt{5} + \sqrt{3}$$

$$\text{c) } \frac{1}{1 + \sqrt{2}} = \frac{1 - \sqrt{2}}{(1 + \sqrt{2})(1 - \sqrt{2})} = \frac{1 - \sqrt{2}}{1 - 2} = \sqrt{2} - 1$$

$$\text{d) } \frac{3 + \sqrt{2}}{\sqrt{2}} = \frac{(3 + \sqrt{2})\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{3\sqrt{2} + 2}{2}$$

$$\text{e) } \frac{\sqrt{2}}{3 - \sqrt{2}} = \frac{\sqrt{2}(3 + \sqrt{2})}{(3 - \sqrt{2})(3 + \sqrt{2})} = \frac{3\sqrt{2} + 2}{9 - 2} = \frac{3\sqrt{2} + 2}{7}$$