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Maths 3rd ESO

EXAM 2_2 (PROGRESSIONS)

Remember: in each question, write the steps you have taken to reach the solution.

1) For each of the following sequences, determine whether it is arithmetic or geometric. Find the ninth term, and the sum of the first 18 terms. (3 points)

a) 6, 1, -4, …

b) 0, 11, 22, …

c) 3, -6, 12,…

2) The second term of a positive geometric progression is 32 and the fourth term is 8. Find the first term and the sum of all its terms. (1.5 p)

3) In an arithmetic progression the first term is 2 and the common difference is 3. The sum of n first term is 222, find n. (1.5 points)

4) The ages of five brothers and sisters form an arithmetic sequence. The youngest is 8 years old and the eldest is 20 years old.
a) What is the sequence's difference?
b) How old are the rest of the brothers and sisters?
c) In ten years, will their ages still form an arithmetic sequence? What is it? (1.5 points)

5) Solve the following equations: (2.5 points)

a) $\frac{3x+1}{7}-\frac{2-4x}{3}=\frac{7x}{6}-\frac{5x+4}{14}$

b) $3(2-x)^2-(3+x)(3-x)=12(1-x)$

SOLUTION

1) For each of the following sequences, determine whether it is arithmetic or geometric. Find the ninth and terms, and the sum of the first 18 terms.

a) 6, 1, -4, ... AP $d=-5,\quad a_1=6\quad a_n=6+(n-1)(-5)$

$a_9=6+8(-5)=6-40=-34$, $a_{18}=6+17(-5)=6-85=-79$

$$S_{18}=\frac{(a_1+a_{18})\cdot 18}{2}=(6-79)\cdot 9=-657$$

b) 0, 11, 22, ... AP $d=11,\quad a_1=2\quad a_n=0+(n-1)11=11n-11$

$a_9=11\cdot 9-11=88$, $a_{18}=11\cdot 18-11=187$

$$S_{18}=\frac{(a_1+a_{18})18}{2}=(0+187)\cdot 9=1683$$

c) 3, -6, 12,... GP $r=-2,\ a_1=3\rightarrow a_n=3\cdot(-2)^{n-1}$

$a_9=3\cdot(-2)^8=3\cdot 256=768$; $a_{18}=3\cdot(-2)^{17}$

$$S_{18}=\frac{3\cdot(-2)^{17}(-2)-3}{-2-1}=\frac{3\cdot 2^{18}-3}{-3}=\frac{3\left(2^{18}-1\right)}{-3}=-2^{18}+1=-262143$$

2) The second term of a positive geometric progression is 32 and the fourth term is 8. Find the first term and the sum of all its terms.

GP $a_2=32,\ a_4=8\rightarrow a_4=a_2\cdot r^2\Rightarrow 8=32r^2\Rightarrow r^2=\frac{1}{4}\Rightarrow r=\frac{1}{2}$

$a_2=a_1\cdot r\Rightarrow 32=a_1\cdot\frac{1}{2}\Rightarrow a_1=64$ The first term is 64

$S=\frac{a_1}{1-r}=\frac{64}{1-\frac{1}{2}}=64\div\frac{1}{2}=128$ The sum of all its terms is 128

3) In an arithmetic progression the first term is 2 and the common difference is 3. The sum of n first term is 222, find n.

AP $a_1=2,\ d=3\rightarrow S_n=222\Rightarrow 222=\frac{(a_1+a_n)\cdot n}{2}\Rightarrow 444=(2+a_n)\cdot n$

$a_n=2+(n-1)3=3n-1\ \rightarrow 444=(2+3n-1)\cdot n\rightarrow 444=n+3n^2$

$$3n^2+n-444=0\Rightarrow n=\frac{-1\pm\sqrt{1+12\cdot 444}}{6}=\frac{-1\pm 73}{6}=\begin{cases}12\\ -\frac{37}{3}\ \text{No}\end{cases}$$

Answer: There are 12 terms

4) The ages of five brothers and sisters form an arithmetic sequence. The youngest is 8 years old and the eldest is 20 years old.

a) What is the sequence's difference?

$a_1 = 8,\ a_5 = a_1 + 4d = 20 \Rightarrow 8 + 4d = 20 \Rightarrow 4d = 12 \Rightarrow d = 3$

b) How old are the rest of the brothers and sisters?

The ages are: 8, 11, 14, 17 and 20

c) In ten years, will their ages still form an arithmetic sequence? What is it?

Yes, in ten years they will be 18, 21, 24, 27, 30 AP of difference 3

5) Solve the following equations:

a) $\frac{3x+1}{7} - \frac{2-4x}{3} = \frac{7x}{6} - \frac{5x+4}{14} \rightarrow \frac{18x+6}{42} - \frac{28-56x}{42} = \frac{49x}{42} - \frac{15x+12}{42}$

$18x + 6 - 28 + 56x = 49x - 15x - 12 \rightarrow 18x + 56x - 49x + 15x = -12 + 28 - 6$

$40x = 10 \Rightarrow x = \frac{1}{4}$

b) $3(2-x)^2 - (3+x)(3-x) = 12(1-x) \rightarrow 3\left(4 - 4x + x^2\right) - \left(9 - x^2\right) = 12 - 12x$

$12 - 12x + 3x^2 - 9 + x^2 = 12 - 12x \rightarrow 4x^2 + 3 = 12 \rightarrow 4x^2 = 9 \rightarrow x^2 = \frac{9}{4}$

$x = \pm\sqrt{\frac{9}{4}} = \pm\frac{3}{2}$