EXAM 3_3 (Coordinate Geometry_Functions)

1. A circle has center at (0,4) and passes through the point (3,0). Find an equation to this circle (1.5 points)

2. A triangle has vertices A(-3,3), B(-3,-2), C(5,-2). Show that it is a right angled triangle. Calculate its perimeter and area. (1.5 points)

3. Find the equation of the line which passes through the origin and the point of intersection of $y = x + 4$ and $x + y = 6$. (1.75 points)

4. Find the equation of the perpendicular bisector of the line segment AB, with A(-1,2) and B(1,4). Find a parallel line to AB, passing through the point P(-3,2). (1.75 points)

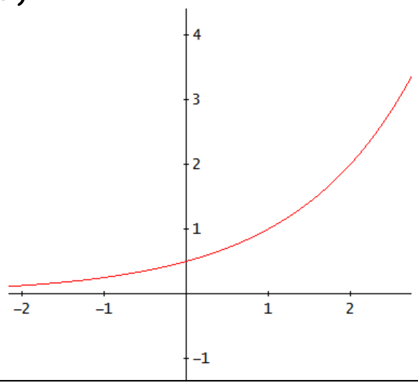
5. A quadrilateral has vertices A(5,4), B(-2,5), C(-1,-2) and D(6,-3). Show that the quadrilateral is a rhombus and calculate the area of ABCD. (1.75 points)

6. Match the equations to the corresponding graphs (explaining your answer):

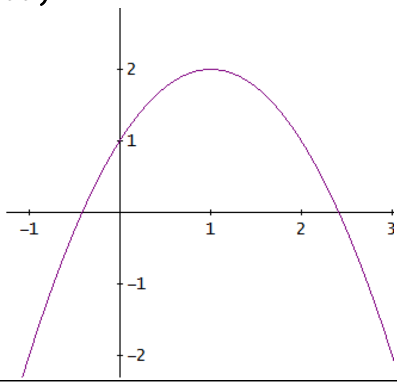
a) $y = -2x^2 + 4x$ b) $y = \dfrac{4}{x+1}$ c) $y = 2^{x-1}$

d) $y = -x^2 + 2x + 1$ e) $y = \log_{\frac{1}{2}} x$ f) $y = -\dfrac{4}{x-1}$

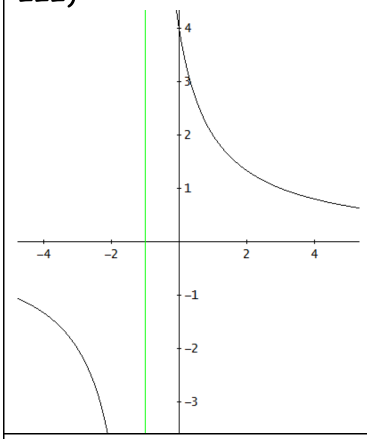
I)



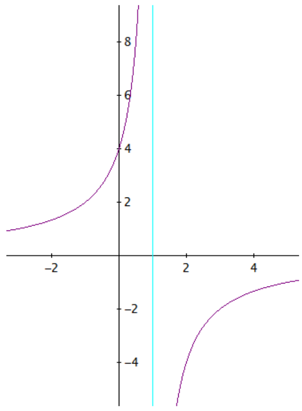
II)



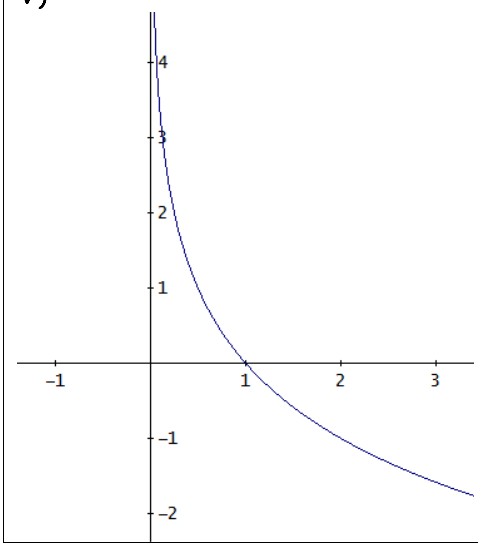
III)



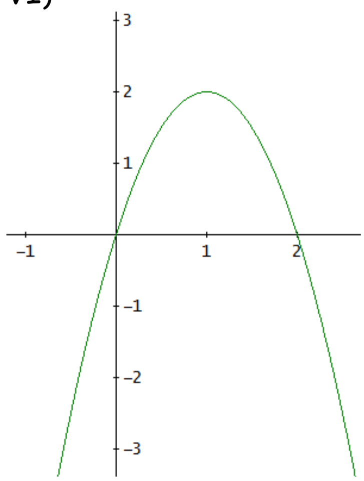
IV



V)

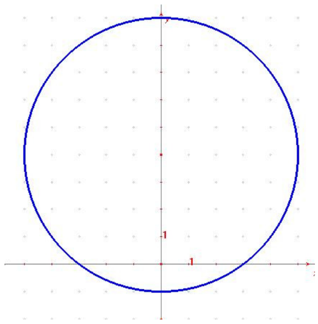


VI)



SOLUTION

1. A circle has center at (0,4) and passes through the point (3,0). Find an equation to this circle

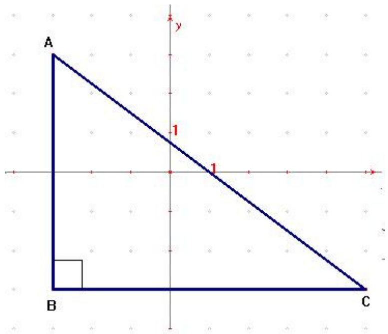


$(x-0)^2+(y-4)^2=r^2$, It passes through the point (3,0)

$\rightarrow 3^2+(0-4)^2=r^2$

$\rightarrow 9+16=r^2 \rightarrow r^2=25 \rightarrow$ Equation: $x^2+(y-4)^2=25$

2. A triangle has vertices A(-3,3), B(-3,-2), C(5,-2). Show that it is a right angled triangle. Calculate its perimeter and area.



If it was a right triangle $\rightarrow$Pythagorean Theorem: $\rightarrow b^2+c^2=h^2$

We are going calculate the distances, to get the legs and hypotenuse:

$d(A,B)=\sqrt{(-3+3)^2+(-2-3)^2}=\sqrt{0+25}=5u$

$d(A,C)=\sqrt{(5+3)^2+(-2-3)^2}=\sqrt{64+25}=\sqrt{89}\,u \rightarrow$ hypotenuse ? the biggest

$d(B,C)=\sqrt{(5+3)^2+(-2+2)^2}=\sqrt{64+0}=8u$

$\overline{AC}^2=\overline{AB}^2+\overline{BC}^2 \Leftrightarrow \left(\sqrt{89}\right)^2=5^2+8^2 \rightarrow 89=25+64 \rightarrow 89=89$

So, yes, it is a right angled triangle.

Perimeter: $P=5+8+\sqrt{89}=13+\sqrt{89}$ u

Area: $A=\dfrac{5\times 8}{2}=20u^2$

3. Find the equation of the line which passes through the origin and the point of intersection of y = x + 4 and x + y = 6 .

Point A (0,0), point B:

$\left.\begin{array}{l} y=x+4 \\ x+y=6 \end{array}\right\} x+x+4=6 \rightarrow 2x=2 \rightarrow x=1$

$y=1+4=5 \rightarrow B(1,5)$

Equation $\overline{AB}$: $\dfrac{x-0}{1-0}=\dfrac{y-0}{5-0} \rightarrow 5x=y \rightarrow y=5x$

4. Find the equation of the perpendicular bisector of the line segment AB, with A(-1,2) and B(1,4). Find a parallel line to AB, passing through the point P(-3,2).

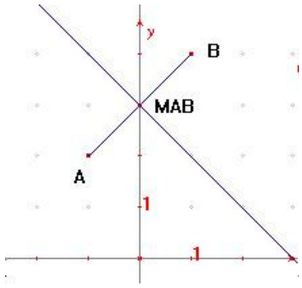
Equation of $\overline{AB}$: $\frac{x+1}{1+1}=\frac{y-2}{4-2} \rightarrow 2(x+1)=2(y-2) \rightarrow x+1=y+2 \rightarrow y=x-1$

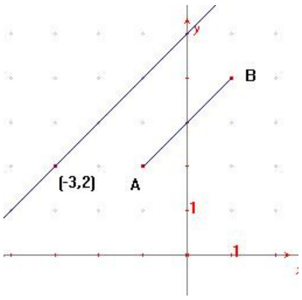
Perpendicular bisector(It passes through the midpoint of AB, perpendicular):

$M_{AB}=\left(\frac{-1+1}{2},\frac{2+4}{2}\right)=(0,3) \rightarrow$ slope:

$m'=-\frac{1}{m}=-\frac{1}{1}=-1$

$y-3=-1(x-0) \rightarrow y=-x+3$ Perpendicular bisector

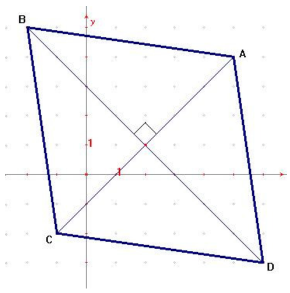




Parallel to AB passing through (-3,2): slope m = 1

$y-2=1(x+3) \rightarrow y=x+5$ parallel line to AB

5. A quadrilateral has vertices A(5,4), B(-2,5), C(-1,-2) and D(6,-3). Show that the quadrilateral is a rhombus and calculate the area of ABCD.



If the quadrilateral is a rhombus, its diagonals are perpendicular lines. We are going to find the equations of the diagonals $\overline{AC}$ and $\overline{BD}$:

$\overline{AC}$: $\frac{x-5}{-1-5}=\frac{y-4}{-2-4} \rightarrow -6(x-5)=-6(y-4) \rightarrow x-5=y-4 \rightarrow y=x-1$

$\overline{BD}$: $\frac{x+2}{6+2}=\frac{y-5}{-3-5} \rightarrow -8(x+2)=8(y-5) \rightarrow -x-2=y-5 \rightarrow y=-x+3$

We study the slopes: $m_{AC}=1, \quad m_{BD}=-1 \rightarrow$ perpendicular, so it is a rhombus.

Area: $A=\frac{D\times d}{2}$

$d=d(A,C)=\sqrt{(-1-5)^2+(-2-4)^2}=\sqrt{36+36}=\sqrt{72}=6\sqrt{2}$ u

$D=d(B,D)=\sqrt{(6+2)^2+(-3-5)^2}=\sqrt{64+64}=\sqrt{128}=8\sqrt{2}$ u

$A=\frac{D\times d}{2}=\frac{8\sqrt{2}\times 6\sqrt{2}}{2}=48u^2$

6. Match the equations to the corresponding graphs (explaining your answer):

a) $y = -2x^2 + 4x$ Parabola $\cap$ passing through (0,0) $\rightarrow$ Graph VI

b) $y = \frac{4}{x+1}$ Hyperbole, asymptote x= -1 $\rightarrow$ Graph III

c) $y = 2^{x-1}$ Exponential , asymptote x-axis, passing (0,1/2) $\rightarrow$ Graph I

d) $y = -x^2 + 2x + 1$ Parabola $\cap$ passing through (0,1) $\rightarrow$ Graph II

e) $y = log_{\frac{1}{2}} x$ Logarithmic, asymptote y-axis, passing (1,0) $\rightarrow$ Graph V

f) $y = -\frac{4}{x-1}$ Hyperbole, asymptote x = 1 $\rightarrow$ Graph IV