1st TERM GENERAL EXAM

Remember: in each question, write the steps you have taken to reach the solution. (1 point each question)

1) Classify according to number type and mark on the real number line the following. (Notice that some numbers may be of more than one type).

a. $-\sqrt{3}$

b. 2.25

c. $+\sqrt{9}$

d. $-1.1\widehat{6}$

2) Calculate and simplify:

a. $2\sqrt[3]{a^6b} - 3a^2\sqrt[3]{64b} + 5a\sqrt[3]{a^3b} + a^2\sqrt[3]{125b}$

b. $\dfrac{\sqrt{4}.\sqrt[3]{36}.\sqrt[6]{3}}{\sqrt{8}}$

3) Rationalise, work out and simplify: $\dfrac{\sqrt{a}}{2-\sqrt{a}} - \dfrac{\sqrt{a}}{2+\sqrt{a}} =$

4) Solve: $2x^4 + x^3 - 8x^2 - x + 6 = 0$

5) Solve: $\sqrt{x+3} - \sqrt{2x-2} = 2$

6) Solve: $\dfrac{1}{x^2 - x} - \dfrac{1}{x-1} = 0$

7) Solve: $2\left(x^4 - 2x^2 - 15\right)\left(2x^2 - 8\right) = 0$

8) Solve the following simultaneous equation: $\left.\begin{array}{l} \dfrac{3}{x} + \dfrac{2}{y} = 1 \\ x \cdot y = 24 \end{array}\right\}$

9) $6000 is divided between two accounts, one paying 4%interest and the other paying 3%interest. At the end of one interest period, the interest earned by the 4% account exceeds the interest earned by the 3% account by $65. How much was invested in each account?

10) The area of a rectangle is 560 square centimetres. The length is 3 more than twice the width. Find the length and the width.

SOLUTION

1) Classify according to number type and mark on the real number line the following. (Notice that some numbers may be of more than one type).

a. $-\sqrt{3}$ Real, Irrational, negative

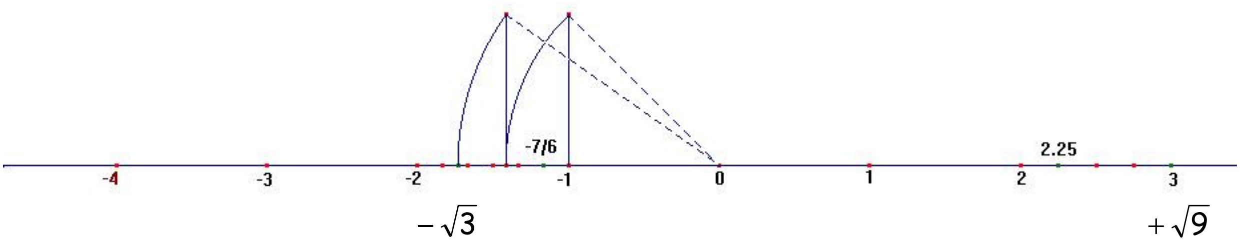
b. 2.25 Real, Rational

c. $+\sqrt{9}$ Real, Rational, Natural (+2)

d. $-1.1\widehat{6}$ Real, Rational, Recurrent decimal, negative

$$2.25 = \frac{225}{100} = \frac{9}{4}; \quad N = 1.1666... \begin{cases} 100N = 116.666... \\ 10N = 11.6666.... \end{cases} \rightarrow 90N = 15 \rightarrow N = \frac{105}{90} = \frac{7}{6}$$

$$-1.1\widehat{6} = -\frac{7}{6}$$



2) Calculate and simplify:

a.

$$2\sqrt[3]{a^6b} - 3a^2\sqrt[3]{64b} + 5a\sqrt[3]{a^3b} + a^2\sqrt[3]{125b} = 2a^2\sqrt[3]{b} - 3a^2\sqrt[3]{2^6b} + 5a\cdot a\sqrt[3]{b} + a^2\sqrt[3]{5^3b} =$$

$$= 2a^2\sqrt[3]{b} - 3a^2\cdot 2^2\sqrt[3]{b} + 5a^2\sqrt[3]{b} + a^2\cdot 5\sqrt[3]{b} = \left(2a^2 - 12a^2 + 5a^2 + 5a^2\right)\sqrt[3]{b} = 0$$

b. $$\frac{\sqrt{4}.\sqrt[3]{36}.\sqrt[6]{3}}{\sqrt{8}} = \frac{\sqrt[6]{\left(2^2\right)^3}\cdot\sqrt[6]{2^4\cdot 3^4}\cdot\sqrt[6]{3}}{\sqrt[6]{\left(2^3\right)^3}} = \frac{\sqrt[6]{2^6\cdot 2^4\cdot 3^4\cdot 3}}{\sqrt[6]{2^9}} = \sqrt[6]{\frac{2^{10}\cdot 3^5}{2^9}} = \sqrt[6]{2\cdot 3^5}$$

3) Rationalise, work out and simplify:

$$\frac{\sqrt{a}}{2-\sqrt{a}} - \frac{\sqrt{a}}{2+\sqrt{a}} = \frac{\sqrt{a}\cdot\left(2+\sqrt{a}\right)}{\left(2-\sqrt{a}\right)\left(2+\sqrt{a}\right)} - \frac{\sqrt{a}\cdot\left(2-\sqrt{a}\right)}{\left(2+\sqrt{a}\right)\left(2-\sqrt{a}\right)} = \frac{2\sqrt{a}+a}{4-a} - \frac{2\sqrt{a}-a}{4-a} = \frac{2a}{4-a}$$

4)

5) Solve: $2x^4 + x^3 - 8x^2 - x + 6 = 0$

Factors of 6: $\pm 1, \pm 2, \pm 3, \pm 6$

$P(x) = 2x^4 + x^3 - 8x^2 - x + 6$

$P(1) = 2\cdot 1^4 + 1^3 - 8 - 1 + 6 = 0;$

	2	+1	- 8	-1	+6
1		+2	+3	-5	-6
	2	+3	- 5	-6	0
-1		-2	-1	+6	
	2	+1	- 6	0	

$Q(x) = 2x^3 + 3x^2 - 5x - 6$
$Q(-1) = -2 + 3 + 5 - 6 = 0$

$$2x^2 + x - 6 = 0 \rightarrow x = \frac{-1 \pm \sqrt{1+48}}{4} = \frac{-1 \pm 7}{4} = \left\langle \begin{matrix} -2 \\ \frac{3}{2} \end{matrix} \right.$$

Solution: $x = 1, x = -1, x = -2, x = \frac{3}{2}$

5) $\sqrt{x+3} - \sqrt{2x-2} = 2 \rightarrow \sqrt{x+3} = 2 + \sqrt{2x-2}$ to the power of two:

$$x + 3 = 4 + 4\sqrt{2x-2} + 2x - 2 \rightarrow 4\sqrt{2x-2} = 1 - x \rightarrow \left(4\sqrt{2x-2}\right)^2 = (1-x)^2 \rightarrow$$
$$\rightarrow 16(2x-2) = 1 - 2x + x^2 \rightarrow 32x - 32 = 1 - 2x + x^2 \rightarrow x^2 - 34x + 33 = 0$$
$$x = \frac{34 \pm \sqrt{1156 - 132}}{2} = \frac{34 \pm 32}{2} \rightarrow \begin{cases} x = 33 \\ x = 1 \end{cases}$$

Check: $\sqrt{33+3} - \sqrt{66-2} = 2 \rightarrow \sqrt{36} - \sqrt{64} = 6 - 8 \neq 2$ It is not a solution
$\sqrt{1+3} - \sqrt{2-2} = 2 \rightarrow \sqrt{4} - \sqrt{0} = 2 - 0 = 2$ It is a solution **Solution: x = 1**

6) $\frac{1}{x^2 - x} - \frac{1}{x-1} = 0$ LCM= $x(x-1) = x^2 - x$

$$\frac{1}{x^2 - x} - \frac{1}{x-1} = 0 \rightarrow \frac{1}{x^2 - x} - \frac{x}{x^2 - x} = \frac{0}{x^2 - x}$$
$$1 - x = 0 \rightarrow x = 1$$

x= 1 It I'snt a solution (denominator 0) No solution

7) Solve: $2\left(x^4 - 2x^2 - 15\right)\left(2x^2 - 8\right) = 0 \begin{cases} x^4 - 2x^2 - 15 = 0 \\ 2x^2 - 8 = 0 \end{cases}$

$$x^4 - 2x^2 - 15 = 0 \rightarrow \text{biquadratic} \rightarrow z = x^2 \Rightarrow z^2 - 2z - 15 = 0$$
$$z = \frac{2 \pm \sqrt{4 + 60}}{2} = \frac{2 \pm 8}{2} = \left\langle \begin{matrix} 5 \\ -3 \end{matrix} \right. \rightarrow \left\langle \begin{matrix} x = \pm\sqrt{5} \\ x = \sqrt{-3} \end{matrix} \right.$$
$$2x^2 - 8 = 0 \rightarrow 2x^2 = 8 \rightarrow x^2 = 4 \rightarrow x = \pm 2$$

Solution: $x = 2, x = -2, x = \sqrt{5}, x = -\sqrt{5}$

8) Solve the following simultaneous equation: $\left.\begin{array}{l}\frac{3}{x}+\frac{2}{y}=1\\ x\cdot y=24\end{array}\right\}$

$$\left.\begin{array}{l}\frac{3}{x}+\frac{2}{y}=1\\ x\cdot y=24\end{array}\right\}\rightarrow\left.\begin{array}{l}3y+2x=xy\\ x\cdot y=24\end{array}\right\}\rightarrow y=\frac{24}{x}\rightarrow 3\cdot\frac{24}{x}+2x=x\cdot\frac{24}{x}$$

$$\frac{72}{x}+2x=24\rightarrow 72+2x^2=24x\rightarrow x^2-12x+36=0\rightarrow x=\frac{12\pm\sqrt{0}}{2}=6$$

$$\rightarrow x=6\Rightarrow y=\frac{24}{x}=\frac{24}{6}=4$$

Solution: $x = 6, y = 4$

9) \$6000 is divided between two accounts, one paying 4%interest and the other paying 3% interest. At the end of one interest period, the interest earned by the 4% account exceeds the interest earned by the 3% account by \$65. How much was invested in each account?

Money invested at 4% -(x) Money invested at 3% - (6000-x)

$$\frac{3}{100}\cdot(6000-x)+65=\frac{4}{100}x\rightarrow 18000-3x+6500=4x\rightarrow 7x=24500\rightarrow x=3500$$

Answer – They invested \$3500 in the 4% account and 2500 in the 3% account.

10) The area of a rectangle is 560 square centimetres. The length is 3 more than twice the width. Find the length and the width.

Length of the rectangle – 2x+3 Width of the rectangle – x

So, $x\cdot(2x+3)=560\Rightarrow 2x^2+3x-560=0\rightarrow$

$$x=\frac{-3\pm\sqrt{9+4480}}{4}=\frac{-3\pm 67}{4}=\left\langle\begin{array}{l}16\\ -\frac{35}{2}\end{array}\right.\rightarrow 2\cdot 16+3=35$$

Answer –The width of the rectangle is 16 cm and its width is 35 cm.